Estimating Central Bank Preferences under Commitment and Discretion

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Abstract
This paper explains US macroeconomic outcomes with an empirical new-Keynesian model in which monetary policy minimizes the central bank’s loss function. The presence of expectations in the model forms a well-known distinction between two modes of optimization, termed commitment and discretion. The model is estimated separately under each policy using maximum likelihood over the Volcker-Greenspan-Bernanke period. Comparisons of fit reveal that the data favor the specification with discretionary policy. Estimates of the loss function weights point to an excessive concern for interest rate smoothing in the commitment model but a more balanced concern relative to inflation and output stability in the discretionary model.

Keywords: Optimal Monetary Policy, Commitment, Discretion, Policy Preferences

JEL Classification: E52, E58, E61, C32, C61
1 Introduction

In models that embody rational expectations, optimal monetary policies are separated by a dichotomy known as commitment and discretion. As first pointed out by Kydland and Prescott (1977), the distinction between them is whether promises made at an earlier time restrict the policy choices of today. A central bank operating under commitment gives assurances about how policy will be set in all future periods through the design of a contingency rule for the nominal interest rate. By a contingency rule I mean one involving instrument settings that are conditional on the state of the economy. The salient aspect of commitment is that policymakers deliver on past promises by responding to economic conditions in accordance with the original plan. A central bank practicing discretion, however, is not bound by some predetermined course of action. Changes in the interest rate are instead the result of period-by-period reoptimization in which foregoing policy intentions are considered irrelevant for current decision making. It follows that measures taken to stabilize the economy do not constrain future policy management in any credible way.

Ever since the distinction between commitment and discretion was first recognized, countless studies have assessed their performance in a variety of macroeconomic models. A consistent theme of this literature is that commitment is the better policy because it generates lower average inflation in the long run (Barro and Gordon 1983, Rogoff 1985) and a more efficient response of the economy to random shocks in the short run (Woodford 1999, Clarida, Galí, and Gertler 1999). The gains from commitment are a direct consequence of the role that expectations play in shaping economic conditions. A policymaker that is understood by private agents to always follow through on promised behavior can harness expectations in a manner that best achieves its objectives. By contrast, exercising pure discretion gives the policymaker less influence over private-sector beliefs and, as a result, produces outcomes that are inferior to commitment.
Given the many advantages of commitment, it is surprising that the empirical literature has had little to say about which of the two policy concepts describes behavior that is closer to how central banks manage interest rates in the real world. Numerous papers analyzing US monetary policy, for example, assume that the Federal Reserve has access to some form of commitment technology, overlooking the possibility that discretion is more compatible with the data. Obscuring the issue further are past statements from leading figures in the policy-making community. In an article summarizing a 2007 speech by Philadelphia Fed President Charles Plosser, Dotsey (2008, p. 8) asserts, “The current Chairman, Ben Bernanke, is maintaining their [Volcker and Greenspan’s] example of commitment to low and stable inflation. The benefits of following a committed plan are now so entrenched in policymaking circles that most central banks aggressively strive to maintain their credibility.” Expressing an alternative view, Bernanke and Mishkin (1997, pp. 105-113) remark that “inflation targeting as it is actually practiced contains a considerable degree of what most economists would define as policy discretion . . . [and] that a major reason for the success of the Volcker-Greenspan Fed is that it has employed a policymaking philosophy, or framework, which is de facto very similar to inflation targeting.” Conflicting anecdotal accounts like these together with insufficient statistical evidence led McCallum (1999, p. 1489) to conclude “that neither of these two modes of central bank behavior—rule-like [commitment] or discretionary—has as yet been firmly established as empirically relevant.”

This paper attempts to bridge the gap in the research on commitment and discretion put forward by McCallum (1999). It starts from the presumption that the Fed sets interest rates in a deliberate fashion with specific goals in mind, and then asks whether it is possible to infer from the data which mode of optimization best explains macroeconomic outcomes in the US. Of course, discriminating commitment-like from discretionary activity over the sample can only be accomplished with an econometric procedure that gives voice to the explicit optimization problem of the policymaker. To that end, this paper borrows from a recent
literature that estimates the parameters of an aggregate demand and supply model subject to the constraint that the policy component minimizes the central bank’s loss function (Ozlale 2003, Favero and Rovelli 2003, Dennis 2006). Conditioning estimation on the requirement that interest rates are chosen optimally, be it under commitment or discretion, enables one to obtain joint estimates of the structural parameters that characterize private behavior and the loss function weights that reveal the preferences of monetary policy.

To determine which one is the more “empirically relevant,” I perform joint estimation separately under commitment and discretion and consider various measures of fit as a way of assessing congruence between the data and the models. The emphasis on relative model fit is appropriate because the two policies impose different cross-equation restrictions on the complete model in equilibrium. Utilizing all pertinent data in a framework that accounts for the particular restrictions implied by commitment or discretion should help identify which policy is more likely to have produced the observed outcomes.

The exercise described above is carried out using a simple new-Keynesian model of output and inflation dynamics. The structural equations form the constraints for the central bank’s optimization problem. The stabilization goals of policy are represented by a quadratic loss function that penalizes deviations of inflation and output from target in addition to changes in the instrument setting. The last argument is often referred to as an interest rate smoothing objective. Structural parameters and loss function weights are estimated simultaneously, once under commitment and once under discretion, using a maximum-likelihood procedure with quarterly US data spanning the chairmanships of Volcker, Greenspan, and Bernanke.

In evaluating the performance of commitment and discretion, I appeal to formal measures of fit provided through the likelihood function as well as informal comparisons of second moments. Regarding the latter, I find that discretion does a better job of matching all the standard deviations calculated from the sample. It also dominates commitment in terms of the autocorrelations for inflation and the cross correlations between the output gap and
inflation. For likelihood-based comparisons I employ the Bayesian information criterion and a related pseudo-posterior odds ratio which summarizes the probability of a model given the available data. I focus on these statistics rather than the raw log-likelihood values because the policies examined in this paper are non-nested. It turns out that the information criterion is considerably higher in the case of discretion, indicating greater fit with the data. The corresponding pseudo-odds measure reports a conditional probability of less than one percent for the commitment model compared to ninety-nine percent for discretion.

Another issue concerns the extent to which commitment and discretion yield different estimates of key parameters, specifically the loss function weights. I find that parameter estimates are similar across policies with one exception. Under commitment the weight on interest rate smoothing is significantly larger than the ones on inflation or the output gap. Under discretion, however, the weight on policy smoothing is estimated to be the smallest of the three, a more plausible result considering the traditional focus among central banks on the other two objectives (i.e., the “dual mandate” of the Federal Reserve). The impulse response functions reveal that this divergence is mainly driven by the propensity for commitment to increase the volatility of the interest rate which, in turn, forces maximum likelihood to lift the smoothing penalty in an attempt to reconcile the model with the data.

The fact that empirical evidence favors discretion raises the question of whether the US economy would have evolved differently had the Fed operated under commitment during the Volcker-Greenspan-Bernanke era. To shed light on this matter, I simulate the model with commitment using the parameter estimates obtained under discretion. The shocks used to generate the counterfactual series are the “true” structural shocks estimated from the discretionary model. Simulation results make clear that while the interest rate would have been more volatile under commitment, the paths of inflation and the output gap would have been close to actual outcomes. As summarized by the loss function, the improvement in macroeconomic stabilization that would have occurred had policy been set under commitment rather
than discretion is equivalent to a permanent shift in inflation of only 0.37 percentage points.

This paper is not the first to estimate the central bank’s loss function. Early examples are Ozlale (2003) and Favero and Rovelli (2003), both of which employ versions of the backward-looking model of Rudebusch and Svensson (1999) and find evidence of a structural break in the policy weights after Volcker’s appointment to chairman of the Federal Reserve. Dennis (2006) uses the same model to estimate the Fed’s implicit inflation target and to see whether the hypothesis of optimal policy can be rejected by the data. This study differs from the early literature in an important way. My model is forward looking, implying a separation between commitment and discretion that is entirely absent in the Rudebusch-Svensson model.


While the papers in this literature deal with a variety of specific issues, they all share one aspect in common. Each one makes an a priori assumption about the nature of monetary policy by considering only one of the two possible styles of optimization. I take a step back and attempt to infer from the data which style is more empirically relevant. To my knowledge, this is the first paper that systematically compares the empirical effects of commitment and discretion by estimating the two policies side-by-side.
2 A Small Empirical Model of the US Economy

The model has three components. The first is an IS equation and a Phillips curve that form the constraints for the central bank’s control problem. The second is a loss function that summarizes the goals of monetary policy. The third component is a procedure for determining the path of the interest rate, namely, optimization under commitment or discretion.

2.1 The Policy Constraints

The constraints belong to a family of new-Keynesian models that have been applied extensively in the study of optimal monetary policy. There is a large literature showing that the behavioral equations comprising these models can be derived from a general equilibrium framework (Kimball 1995, Yun 1996, Rotemberg and Woodford 1997, McCallum and Nelson 1999). The policy implications of a purely forward-looking class of new-Keynesian models are examined by Clarida et al (1999). The version used here augments the forward-looking specification with backward-looking elements designed to capture persistent aspects of the data (Fuhrer and Moore 1995, Fuhrer 1997, Estrella and Fuhrer 2002).

Denote $y_t$ the output gap, the log deviation of real output from potential, and let $\pi_t$ be the inflation rate between dates $t-1$ and $t$. The output gap is determined by an IS equation

$$y_t = \phi E_t y_{t+1} + (1 - \phi)(\beta y_{t-1} + (1 - \beta)y_{t-2}) - \sigma (i_t - E_t \pi_{t+1}) + u_{y,t},$$

(1)

where $i_t$ is the (short-term) nominal interest rate, and $E_t$ is an expectations operator conditional on date-$t$ information. When $\phi = 1$, (1) corresponds to the log-linearized Euler equation describing the household’s optimal consumption plan. The parameter $\sigma$ can be interpreted in this case as the intertemporal elasticity of substitution. The exogenous variable $u_{y,t}$ is a mean-zero, serially uncorrelated demand shock with constant variance $\sigma_y^2$. 
The presence of lagged output in the IS equation whenever $\phi < 1$ is motivated by empirical concerns. Estrella and Fuhrer (2002) and Fuhrer and Rudebusch (2004) assert that multiple lags are needed to explain the inertial responses of output observed in the data. Including lagged terms is also not inherently at odds with economic theory. Fuhrer (2000) shows that persistence follows directly from first principles when the primitive model exhibits habit formation in consumption. An IS equation with the same lead-lag structure as (1) can be derived from a model with external habit formation (Smets and Wouters 2003, Ravn, Schmitt-Grohé, and Uribe 2006) that depends on on exactly two lags of consumption.\footnote{A longer working paper available from the author contains an appendix that formally derives this result.}

The inflation rate is governed by an expectations-augmented Phillips curve

\[ \pi_t = \alpha E_t \pi_{t+1} + (1 - \alpha) \pi_{t-1} + \kappa y_t + u_{\pi,t}, \tag{2} \]

which relates inflation to past and expected future inflation and the current output gap. For the case of $\alpha = 1$, (2) resembles the “new Phillips curve” estimated by Galí and Gertler (1999). The new Phillips curve is consistent with a model of monopolistically competitive firms that stagger prices according to Calvo (1983). The key cyclical factor affecting pricing decisions is real marginal cost, which can be shown to vary proportionately with the output gap (Woodford 2003a, Ch. 3).\footnote{Galí and Gertler (1999) present evidence showing that synthetic gap variables used in many studies (i.e., deterministic trends, capacity utilization, or the Congressional Budget Office estimate of potential GDP) are poor proxies for the theoretically consistent measure of marginal cost based on the labor income share.} The slope coefficient $\kappa$ carries information regarding the frequency of price revisions. Greater nominal rigidity, meaning less frequent revisions, implies a smaller value of $\kappa$. The variable $u_{\pi,t}$ is a mean-zero, serially uncorrelated supply shock with variance $\sigma_{\pi}^2$. It is often interpreted as a “cost-push” shock reflecting variations in marginal cost unrelated to the output gap (Clarida et al 1999). I allow for nonzero correlation between supply and demand shocks and denote their covariance $\sigma_{y\pi}$.

To account for the degree of persistence found in the data, the Phillips curve includes a
lagged term whenever $\alpha < 1$. Fuhrer and Moore (1995), Fuhrer (1997), and Roberts (1997) argue that without lagged dependence, models produce “jump” dynamics that contradict evidence suggesting inflation responds sluggishly to economic shocks. The presence of a backward-looking term can be motivated in theory by assuming that prices charged by non-optimizing firms are indexed to past inflation. Christiano, Eichenbaum, and Evans (2005) provide an example of full indexation, whereas Smets and Wouters (2003) allow for partial indexation. Alternatively, lagged inflation can result from the existence of firms that use a rule-of-thumb for setting prices that depends on the history of competitors’ prices (Galí and Gertler 1999). Although it is not generally implied by partial indexation or rule-of-thumb behavior, restricting the coefficients on lagged and future inflation to sum to one ensures that (2) is conformable with the view that policy has no long-run effect on output.

### 2.2 The Loss Function

The central bank sets the path of the nominal interest rate to minimize the loss function

$$L_t = E_t(1 - \delta) \sum_{j=0}^{\infty} \delta^j \{\pi^2_{t+j} + \lambda_y y^2_{t+j} + \lambda_{\Delta i}(i_{t+j} - i_{t+j-1})^2\}, \quad (3)$$

where the discount factor $\delta \in (0, 1)$ and $\lambda_y, \lambda_{\Delta i} \geq 0$. The first two terms penalize squared deviations of inflation and output from their respective targets. The inflation target is assumed to be constant over time and, without loss of generality, is normalized to zero. The target for output is the potential level.\(^3\) The third term penalizes deviations of the interest rate from its previous level and is viewed as an interest rate smoothing incentive for the policymaker. The weights $\lambda_y$ and $\lambda_{\Delta i}$ measure the relative preference for stabilizing output and the interest rate smoothing argument.\(^4\) The weight on inflation is normalized to one.

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\(^3\)The model is equivalent to one that accounts for a nonzero inflation target, but where the variables appearing in (1) and (2) are written as deviations from target values. See Dennis (2004) for an illustration.

\(^4\)The phrase “interest rate smoothing” is used to denote an explicit preference for reducing the variance of the interest rate in first differences. There are others who use the term to mean a reaction function in
The loss function (3) is an appealing way to model central bank preferences for many reasons. First, versions of (3) are commonly used to assess the performance of policy rules (Rudebusch and Svensson 1999, Levin and Williams 2003). As a result, estimates of $\lambda_y$ and $\lambda_{\Delta i}$ have a familiar interpretation and are comparable to the literature on optimal monetary policy. Second, Svensson (1999) argues that the main objectives of a flexible inflation-targeting central bank can be described with a loss function that stabilizes inflation and a measure of real activity. This point is particularly relevant because many have argued that the Federal Reserve under Volcker and Greenspan employed a policy framework that closely resembles inflation targeting (Bernanke and Mishkin 1997, Goodfriend 2003). Third, Woodford (2002) shows that a loss function similar to (3), but without interest rate smoothing, is proportional to a quadratic approximation of the representative consumer’s expected utility, so it provides a natural welfare criterion for ranking alternative policies.

Despite its theoretical appeal, one implication of using a strict, utility-based measure of loss is that the weights attached to the objectives would depend on the underlying parameters, notably the average frequency of price adjustments. By contrast, this paper regards loss function weights as free parameters that are to be estimated alongside the coefficients in the policy constraints. For this reason, and because it also contains an interest rate argument, the objective function (3) should not be interpreted as an approximation of expected utility.

Finally, including an interest rate smoothing term is empirically compelling because it helps capture the degree of policy gradualism observed in the data. There are many explanations for why gradualism is desirable. Woodford (2003b) shows that interest rate inertia is a defining feature of an optimal inflation-targeting rule. Brainard (1967) demonstrates that interventions should be cautious so as to avoid volatility resulting from misperceptions of a model with parameter uncertainty. Orphanides (2003) argues that similar caution is advisable when there is uncertainty regarding the accuracy of incoming data. Lowe and Ellis which the coefficient on the lagged policy rate is larger than zero (Levin, Wieland, and Williams 1999).
(1997) point out that such preferences may reflect a concern for financial market stability.

### 2.3 Optimal Monetary Policy

To compute optimal policies, stack the constraints in companion form as

\[
\begin{bmatrix}
X_{t+1} \\
\Omega E_t x_{t+1}
\end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + B i_t + \begin{bmatrix} \Gamma u_{t+1} \\ 0_{2 \times 1} \end{bmatrix},
\]

(4)

where \(X_t = [u_{p,t} \ u_{x,t} \ y_{t-1} \ y_{t-2} \ \pi_{t-1} \ i_{t-1}]'\) are the predetermined variables, \(x_t = [y_t \ \pi_t]'\) are the forward-looking variables, and \(u_{t+1} = [u_{p,t+1} \ u_{x,t+1}]'\) are the innovations with covariance matrix \(\Sigma_{uu}\). Structural parameters appear as elements of \(\Omega\), \(A\), and \(B\). The methods in Söderlind (1999) are used to solve for the equilibrium under commitment and discretion.\(^5\)

The central bank selects an interest rate path to minimize (3) subject to (4). Because the model is forward looking, it faces constraints that depend on expectations about the current and future course of monetary policy. In this kind of environment, the ability to credibly commit to a particular sequence of actions has major implications for the economy.

Under commitment the central bank announces at a specific date a complete, state-contingent plan for the interest rate that is to be strictly followed in all subsequent periods. When determining the path of optimal policy, it takes into account how the promise to execute such a contingency plan impacts private-sector expectations. In other words, the central bank internalizes the effect of its decisions on future variables in solving the optimization problem. Commitment thus presumes an ability to fulfill past promises and an understanding on the part of private agents of a willingness to do so regardless of what events transpire. This strategic interaction produces an optimal equilibrium in which the central bank makes efficient use of private-sector beliefs to achieve the goals embodied by the loss function.

\(^5\)Directions on how to construct \(\Omega\), \(A\), \(B\), \(\Sigma_{uu}\), and the selector matrix \(\Gamma\) can be found in the working paper available upon request. It also provides details on the solution method for commitment and discretion.
It can be shown that the equilibrium law of motion under commitment follows

\[
\begin{bmatrix}
X_{t+1} \\
\psi_{t+1}
\end{bmatrix}
= M_c
\begin{bmatrix}
X_t \\
\psi_t
\end{bmatrix}
+ \begin{bmatrix}
\Gamma u_{t+1} \\
0_{2\times1}
\end{bmatrix},
\]  
(5)

\[
\begin{bmatrix}
x_t \\
i_t
\end{bmatrix}
= G_c
\begin{bmatrix}
X_t \\
\psi_t
\end{bmatrix},
\]  
(6)

where \(\psi_t\) are the Lagrange multipliers associated with the lower block of (4). Woodford (2003a, Ch. 7) explains that the multipliers capture the effect of expectations about the current policy setting that are reflected in the decisions private agents have made in all previous periods. It follows that actions taken by the central bank at any given time will depend on the full history of the economy dating back to the policy’s inception.\(^6\)

Under discretion the central bank is free to adjust policy in response to prevailing conditions, but that response does not have to be the one dictated by some contingency rule designed earlier. Instead, a discretionary optimizer evaluates the current and prospective state of the economy and sets policy optimally on the basis of this assessment alone. It repeats the procedure each time an action is considered without making commitments about future policy. Because it cannot shape private expectations in the absence of commitment, future variables are taken as given in the optimization problem. The resulting equilibrium is only optimal in a constrained sense because the central bank, through sequential reoptimization, fails to harness expectations in a way that advances its stabilization goals.

\(^6\)To see this formally, note that one can solve for \(\psi_t\) in terms of \(\{X_j\}_{j=0}^{t-1}\) by inverting the lag polynomial implied by the lower block of (5). Substituting the resulting expression into (6) gives a policy equation that describes the interest rate as a function of all current and past realizations of the predetermined variables.
The equilibrium law of motion under discretion is given by

$$X_{t+1} = M_d X_t + \Gamma u_{t+1}, \quad (7)$$

$$\begin{bmatrix} x_t \\ i_t \end{bmatrix} = G_d X_t. \quad (8)$$

An important characteristic of the policy equation embedded in (8), and one that distinguishes it from commitment, is that it is purely forward looking rather than history dependent. A forward-looking policy is one in which outcomes are determined solely by the current and expected future outlook for the state of the economy. In equilibrium the interest rate depends only on today’s predetermined variables since conditional expectations are computed within the model as linear projections onto the current state.

3 Estimation Strategy

The recursive equilibrium under commitment or discretion takes the form of an empirical state-space model that can be estimated with maximum likelihood using the Kalman filtering algorithms described in Hamilton (1994, Ch. 13). The state equation is

$$\xi_{t+1} = F \xi_t + \tilde{\Gamma} u_{t+1}, \quad (9)$$

with $\xi_t \in \{[X_t' \psi_t]', \ X_t\}$ and $F \in \{M_c, \ M_d\}$. The observation equation is

$$\begin{bmatrix} y_t^o \\ \pi_t^o \\ \bar{i}_t^o \end{bmatrix} = H \xi_t + \begin{bmatrix} 0 \\ 0 \\ v_{i,t} \end{bmatrix}, \quad (10)$$

12
where \( \{y_t^o, \pi_t^o, \iota_t^o\}_{t=1}^T \) denotes the observed output gap, inflation, and the nominal interest rate, and \( H \in \{G_c, G_d\} \). The variable \( v_{i,t} \) is a mean-zero, serially uncorrelated shock to \( \iota_t^o \). Its variance \( \sigma_i^2 \) measures the discrepancy between the optimal policy implied by the model and the actual interest rate in the sample. With data on three variables, adding \( v_{i,t} \) also helps prevent stochastic singularity (Ingram, Kocherlakota, and Savin 1994).

Structural parameters are estimated with US data over the period 1982:Q1 - 2008:Q4. The output gap is the log deviation of real GDP from the Congressional Budget Office estimate of potential GDP. Inflation is the annualized first difference of the log of the GDP implicit price deflator. The interest rate is the annual yield on 3-month Treasury bills.

Prior to estimation the inflation and interest rate series are de-meaned so that their sample averages match the mean values of the corresponding data generated by the model. Because no intercept terms appear in (1) or (2), the model describes a mean-zero process for all variables in the system. De-meaning also implies that the target levels for inflation and the interest rate implicit in the loss function (3) are equal to the averages taken from the data. This is useful for two reasons. First, the objective is to compare model fit and estimates of the loss function weights under commitment and discretion rather than obtain estimates of the Fed’s latent target values. Second, de-meaning ensures that the inflation and interest rate targets are the same for each policy considered during estimation.

The Kalman filter typically begins with a date-0 estimate of the initial state, call it \( \hat{\xi}_{1|0} \), equal to its long-run mean. The mean value consistent with (9) is zero. This raises a potential concern for estimation under commitment because \( \xi_t \) contains the Lagrange multipliers tied to the forward-looking variables. Starting the recursion with \( \psi_t = 0 \) implies that policy decisions are unconstrained in the initial period, so actions taken in that period do not have

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7 Following Dennis (2006), the sample begins in 1982:Q1 in order to exclude the period when the Federal Reserve’s operating procedure focused on targeting the quantity of non-borrowed reserves.

8 This approach has been used by Ozlale (2003), Söderström et al (2005), and Castelnuovo (2006). Studies that estimate the inflation target directly include Favero and Rovelli (2003), Dennis (2004), Dennis (2006), Ireland (2007), and Fève, Matheron, and Sahuc (2010).
to confirm private-sector expectations that were formed at dates preceding the sample. As a result, the central bank will find it optimal to implement the discretionary policy just once in the first period while promising to behave in a committed fashion thereafter. This aspect of commitment, known as time inconsistency, is fundamental to the control of forward-looking systems (Kydland and Prescott 1977), but it is problematic for estimation due to the arbitrary significance it places on the first observation. By initializing the Kalman filter with $\psi_t = 0$, the model’s interpretation of past events would be one in which the Fed ignored commitments made prior to 1982:Q1 while committing to a new plan that was optimal from that specific date onward. This could be viewed as a shortcoming since there is no compelling historical evidence to indicate that such a regime change took place on that particular date.

In estimating the commitment model, it is better to assume that the Fed has announced its contingency rule at a point predating the sample. It follows that the economy’s initial evolution will be consistent with policy actions taken at all dates after the starting period. This is equivalent to adopting an equilibrium concept that relates closely to what Woodford (2003a, Ch. 7) calls the “timeless perspective” policy. Such a policy requires that central bank actions always validate previously-formed expectations even in the initial period. In practice it is found by substituting out the Lagrange multipliers from the first-order conditions of the policymaker’s control problem, yielding a targeting criterion that must be satisfied every period (Giannoni and Woodford 2005, Dennis 2010).

By contrast, I allow the multipliers to enter the model, but I initialize the Kalman filter with nonzero values so that policy does not arbitrarily deviate from the commitment program at the beginning of the sample. The implementation of commitment during estimation is more similar to the policy examined in Khan, King, and Wolman (2003). To make the commitment problem recursive and the resulting policy time consistent, the authors treat all forward-looking constraints as strictly binding in the initial period. This requires augmenting the Lagrangian with a set of lagged multipliers, one for each new constraint imposed on the
optimization problem. The inclusion of auxiliary multipliers makes the first-order conditions time invariant, implying the same policy behavior at all dates over the planning horizon.

Although some elements of $\xi_t$ are unobservable, an informed decision about their starting values can be made using the Kalman filter. The strategy employed here is to estimate the model in a first stage by setting $\hat{\xi}^{(1)}_{1|0} = 0$, at which point the Kalman filter is used to generate a sequence of updated projections of the state $\{\hat{\xi}^{(1)}_{t|t} \}_{t=1}^{T}$. The model is then re-estimated taking as the initial state the mean value of forecasts computed in the previous step, that is, $\hat{\xi}^{(2)}_{1|0} = (1/T) \sum_{t=1}^{T} \hat{\xi}^{(1)}_{t|t}$. This process is repeated until the initial state equals the average forecast, or $\hat{\xi}^{(i+1)}_{1|0} = \hat{\xi}^{(i)}_{1|0}$. Convergence always occurred in less than 10 iterations.9

4 Empirical Results

4.1 Maximum-Likelihood Estimates

Table 1 shows point estimates and standard errors of 11 structural parameters.10 The left panel presents estimates for the commitment case, and the right panel for discretion. The standard errors are the square roots of the diagonal elements of the inverse Hessian matrix.

There are numerous similarities in the estimates across policies. Looking first at the covariances, $\sigma_y$ and $\sigma_x$ indicate that supply shocks are more than twice as volatile as demand shocks. Estimates of $\sigma_i$ suggest that the disparity between optimal and observed interest rates are largely invariant to the two modes of central bank behavior.

Turning to the IS and Phillips curves, estimates of $\phi$ and $\alpha$ indicate that forward and backward-looking terms are important. The estimate of $\phi$ is close to one-third under commitment and about 0.37 under discretion. Both are in the neighborhood of values reported

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9 An alternative strategy adopted by Ilbas (2010) is to partition the sample to include an initialization period that precedes estimation. She finds that a presample period of 20 quarters is sufficient to eliminate any effects on parameter estimates of setting the multipliers equal to zero in the initial period.

10 The central bank’s discount factor $\delta$ is set equal to 0.99 prior to estimation.
by Fuhrer and Rudebusch (2004) and Lindé (2005). The estimates of $\alpha$ are 0.49 and 0.62, echoing Roberts (2005) and Kiley (2007). Estimates of $\kappa$ are small but within the range typical of the literature. Values of $\kappa$ near zero are evidence of long duration of price stickiness or large strategic complementarities with modest price rigidity.\footnote{See Woodford (2003a, Ch. 3) for details.}

Regarding the loss function, estimates of $\lambda_y$ point to a small preference for output gap stability. There is some empirical work suggesting that the Fed demonstrates little concern for the output gap as an independent policy goal. Salemi (2006), for example, finds that the relative weight on output stabilization is only 0.0012 and not significantly different from zero. Dennis (2004) reports estimates equal to zero for both the Volcker-Greenspan period and for the subperiod covering only Greenspan’s chairmanship. Using the Rudebusch-Svensson model, Dennis (2006) also finds that $\lambda_y$ is not significantly different from zero.\footnote{A Wald test of $\lambda_y = 0$ cannot be rejected under discretion. The appendix contained in the working paper version provides a detailed explanation for this finding.}

To see if the output gap weight is significant in the present model, I conduct likelihood ratio tests of the null hypotheses that $\lambda_y = 0$. The first two columns of Table 2 display estimates and log-likelihood values for commitment and discretion when $\lambda_y$ is fixed at zero. Comparing Tables 1 and 2 reveals that the parameter estimates are not greatly affected by the absence of an output objective. Despite these similarities, the likelihood ratio statistics are 4.18 (p-value is 0.041) in the commitment case and 8.14 (p-value is 0.004) under discretion. The hypothesis that $\lambda_y = 0$ is therefore rejected by the data under both policies.\footnote{These findings confirm results in Favero and Rovelli (2003), Ozlale (2003), and Ilbas (2012) showing that the weight on output volatility is small but statistically significant.}

Returning to Table 1, estimates of $\lambda_{\Delta i}$ make clear that the preference for interest rate smoothing is policy dependent. Under commitment $\lambda_{\Delta i} = 2.56$, making interest rate smoothing the leading policy goal followed by inflation and then output stability. This result is similar to Dennis (2004), Dennis (2006), and Söderström et al (2005) who contend that...
optimal and historical policy can be reconciled if interest rate smoothing is the dominant objective in the Fed’s loss function. These authors obtain estimates of $\lambda_{\Delta i}$ ranging from 1.11 to 4.52 over the Volcker-Greenspan era. A very different outcome emerges under discretion, where the estimate of $\lambda_{\Delta i} = 0.06$ places the interest rate objective just below output in the central bank’s ordering of policy goals. Studies that find evidence of a weak desire for policy smoothing are Favero and Rovelli (2003), Castelnuovo and Surico (2004), and Ilbä (2012).

To evaluate the significance of interest rate smoothing, I re-estimate the model subject to constraints on $\lambda_{\Delta i}$ that vary according to the policy. In the commitment case I restrict $\lambda_{\Delta i} = 1$. A test of this restriction is a test of the hypothesis that inflation and interest rate smoothing receive the same weight in the Fed’s loss function. In the discretion case I set $\lambda_{\Delta i} = 0.01$. While my intent was to set $\lambda_{\Delta i} = 0$, preliminary estimation failed because the error covariance matrix of the data became rank-deficient. Setting $\lambda_{\Delta i} = 0$ ensures that policymakers fully insulate the economy from demand shocks because there is no penalty for adjusting the interest rate. With supply shocks as the only source of exogenous variation, the model implies an exact deterministic relationship between the output gap and inflation.\footnote{Due to numerical inaccuracies in computing log likelihood, values of $\lambda_{\Delta i}$ close to zero can produce a covariance matrix that is nearly singular. I found that $\lambda_{\Delta i} = 0.01$ was small enough to form inferences about the statistical contribution of $\lambda_{\Delta i}$, but not so small as to risk encountering stochastic singularity.}

The final two columns of Table 2 display results for the constraints on $\lambda_{\Delta i}$ described above. Setting $\lambda_{\Delta i} = 1$ under commitment has significant effects on the other parameters. Estimates of $\sigma_y$, $\phi$, and $\sigma$, for example, point to larger demand shocks, a diminished role for expected future output in the IS equation, and a stronger real interest rate channel. Given the size of the standard errors, however, much uncertainty remains about their true values. Imposing $\lambda_{\Delta i} = 1$ also lowers maximized log likelihood from $-387.77$ to $-396.56$, producing a likelihood ratio statistic equal to 17.57 ($p$-value < 0.001). The hypothesis that inflation and interest rate smoothing receive the same weight is therefore rejected under commitment. By contrast, fixing $\lambda_{\Delta i} = 0.01$ under discretion has little impact on parameter estimates and
only changes log likelihood from $-379.61$ to $-379.68$. The likelihood ratio statistic is $0.16$ ($p$-value is 0.694), so the hypothesis that $\lambda_{\Delta_i}$ is close to zero cannot be rejected.

### 4.2 Model Comparison

This section compares the empirical performance of commitment and discretion using two criteria. First, a group of second moments are calculated from the data and compared to ones generated by the models to see how well each policy captures key business cycle features. A second comparison is made by appealing to broad measures of fit provided through the likelihood function, namely, the Bayesian information criterion and a corresponding posterior odds measure that reveals the probability of a model given the data.

Table 3 presents the standard deviations of inflation, the output gap, and the interest rate as implied by the data and the models. The discretionary model does a better job of accounting for the standard deviations of all three. The volatility of inflation, in particular, is only slightly larger than the realized volatility in the data. In comparison, the commitment model significantly overstates inflation and output gap volatility.

Figure 1 plots vector autocorrelation functions.\(^{14}\) Most of the correlations produced by the models match their counterparts from the data. For example, both policies deliver substantial output persistence as measured by correlations between current and lagged output gaps. The half-life of this autocorrelation is about 5 quarters. The models also account for the positive and declining lead-lag relationship between inflation and the interest rate. Nevertheless, there are at least two areas where discretion generates an improvement in fit. The first is the degree of inflation persistence. The half-life of these autocorrelations is 2 quarters under discretion but 5 quarters under commitment. Discretionary policy also improves the accuracy of the correlations between the output gap and inflation, as it correctly predicts the sign and magnitude of this relationship at leads and lags of up to one year.

\(^{14}\)The autocorrelations for the data are computed from a fourth-order vector autoregression.
Another way to assess model fit is with the Bayesian information criterion. The BIC is a consistent model-selection criterion that penalizes likelihood by an amount that increases with the number of estimated parameters. An advantage of the BIC is that it facilitates a comparison among non-nested models. Optimal commitment and discretion are non-nested policies since neither one can be obtained by imposing parametric restrictions on the other.

The BIC can also be used to form a pseudo-posterior odds ratio that gives the data-determined probability of a model. Kiley (2007) explains that in large samples the BIC approximates the marginal likelihood of a model in which the data, summarized by the likelihood function, predominates the Bayesian prior distribution of the parameters. A pseudo-odds measure is then formed by replacing marginal likelihood with the BIC in the ratio

\[ \rho(j) = \frac{\exp(BIC(j))}{\sum_{h=1}^{z} \exp(BIC(h))} , \]

where \( \rho(j) \) is the conditional probability of model \( j \) among the \( z \) different models considered. While consistent with a Bayesian approach to model selection, the pseudo-odds ratio is determined solely by the quality of the model’s characterization of the data and not by any prior information concerning the parameter or model space. This follows from the implicit use of equal prior model probabilities in the construction of \( \rho(j) \) and from the BIC being invariant to priors over the parameters within each model.

Table 4 reports log likelihood, the BIC, and the pseudo-posterior odds ratio for the models under commitment and discretion. The BIC is \(-413.47\) for commitment but \(-405.31\) for discretion. As a result, the pseudo-odds statistic points to a very small conditional probability of 0.0003 in the commitment model compared to 0.9997 under discretion.

Table 4 reports log likelihood, the BIC, and the pseudo-posterior odds ratio for the models under commitment and discretion. The BIC is \(-413.47\) for commitment but \(-405.31\) for discretion. As a result, the pseudo-odds statistic points to a very small conditional probability of 0.0003 in the commitment model compared to 0.9997 under discretion.

The evidence in favor of discretion leaves open the question of whether Federal Reserve

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15. The BIC for model \( j \) is \( \ln \mathcal{L}(j) - (N(j)/2) \ln(T) \), where \( \ln \mathcal{L}(j) \) is maximized log likelihood, \( N(j) \) is the number of estimated parameters in model \( j \), and \( T \) is the sample size.

16. Studies that utilize the BIC include Brock, Durlauf, and West (2003), Kiley (2007), and Keen (2009).
behavior was optimal during the sample period. Comparisons of fit between commitment and discretion alone are insufficient to answer this question because both models assume that policy is set optimally, the only difference being in how policymakers manage expectations. To arrange a valid test of the optimal-policy hypothesis, the comparison group must include an encompassing model that does not constrain central bank actions to be the outcome of loss minimization. To that end, I re-estimate (1) and (2) jointly with an unrestricted policy equation that attaches separate response coefficients to the predetermined state variables. The discretionary model is a special case of this three-equation system that results from conditioning estimation on the assumption that response coefficients minimize expected loss.

Relaxing the coefficient restrictions implied by optimal discretion results in

\[ i_t = 1.8651 u_{yt,t} + 0.1389 u_{\pi,t} + 0.5533 y_{t-1} - 0.4836 y_{t-2} + 0.2982 \pi_{t-1} + 0.8821 i_{t-1}, \]  
(11)

where standard errors (in parentheses) are found using the delta method. Since the models are nested, one can test the hypothesis of optimal discretion using the likelihood ratio statistic. Log likelihood in the model with the unrestricted policy is \(-371.35\) and is reported in the third row of Table 4. This model has 15 free parameters, 9 structural parameters plus 6 policy-rule coefficients. The discretionary model, which returns a log likelihood of \(-379.61\), has the same 9 structural parameters but only 2 free parameters in the loss function. It follows that optimal policy places 4 restrictions on the coefficients estimated in (11). The likelihood ratio statistic in this case is 16.51 (\(p\)-value < 0.001), so the data reject the hypothesis that historical outcomes were the result of discretionary optimization.

Pseudo-Bayesian analysis leads to a different conclusion about which policy fits best. The
BIC for the unrestricted model is \(-406.40\), which is smaller than the value for discretion but larger than the one for commitment. When all three are considered, the implied pseudo-posterior odds indicate a conditional probability of 75 percent for the discretionary model but only 25 percent for the unrestricted model. The probability of commitment is near zero. The pseudo-odds criterion therefore points to discretion as the preferred model.\(^{17}\)

### 4.3 The Role of Interest Rate Smoothing

Inferences about the role of interest rate smoothing as an independent stabilization goal vary greatly depending on the policy. Estimates under commitment suggest that it is the most important objective in the Fed’s loss function, but discretion implies that it is the least. This section provides intuition for why maximum likelihood produces contradictory findings.

Figure 2 graphs the responses to demand and supply shocks for two versions of the model. The first version is the estimated model under discretion, and the second takes the same parameter values (including loss function weights) but assumes a policy of commitment. As expected, commitment leads to more stable output and inflation dynamics. After a demand shock both policies generate “hump-shaped” movements in the output gap and inflation, but the amplitude and persistence are smaller under commitment. Inflation jumps under both policies following a supply shock, but mean reversion is more gradual under discretion. It is easy to see how the central bank achieves greater stability by examining the interest rate profile. By promising to keep rates elevated for an extended time, policymakers reduce expected future output gaps and inflation and, consequently, dampen their adjustment in the near term. The only exception is in the response of output to a supply shock, in which case the period of high interest rates causes a sustained drop in output below potential.\(^{18}\)

\(^{17}\)The conflicting evidence on model fit provided by the likelihood ratio test and the pseudo-odds criterion is a consequence of the correction for degrees of freedom accounted for in the BIC.

\(^{18}\)The scale of this departure is much smaller than the one induced by a demand shock, so overall volatility of the output gap is still lower under commitment.
The problem with the commitment outcome from an empirical perspective is that it implies a level of interest rate volatility that is at odds with the data. Any data-fitting exercise will seek parameter values that drive down this volatility. Figure 3 illustrates some of the tradeoffs faced by maximum likelihood when locating an estimate of $\lambda_{\Delta i}$. The figures plot standard deviations of the observables for a range of values of $\lambda_{\Delta i}$, holding the other parameters fixed at their point estimates. The left panel corresponds to the discretionary model and the right panel to the commitment model. Vertical lines indicate the estimates of $\lambda_{\Delta i}$, and crosses identify the sample moments reported in the first column of Table 3.

In the case of discretion, raising $\lambda_{\Delta i}$ lowers the standard deviation of the interest rate but has little effect on output and inflation. Most of the reduction occurs at small values of $\lambda_{\Delta i}$. Under commitment a much bigger weight is needed to reconcile the model with the data. For small values of $\lambda_{\Delta i}$, the interest rate is twice as volatile as the actual series. Increasing $\lambda_{\Delta i}$ causes this standard deviation to fall but those of the output gap and inflation to rise. Naturally, maximum likelihood compromises between moments by selecting a value of $\lambda_{\Delta i}$ at which the volatility of all three exceed their sample counterparts by nontrivial amounts.

Figure 4 graphs impulse responses for the models estimated with commitment and discretion. In contrast to Figure 2, changes in dynamics are now driven by the policy specification as well as variation in all the parameters. As a result of the large estimate of $\lambda_{\Delta i}$, the policy responses to demand and supply shocks under commitment are much closer to the paths observed under discretion. Greater concern for policy smoothing, however, leads to an increase in the volatility of output and inflation. For example, the peak effect of a demand shock on inflation is twice as large for commitment and occurs three quarters later than discretion. Supply shocks also cause inflation to rise in both models, but the adjustment back to steady state is now more gradual under commitment. The output gap responses exhibit a similar pattern of increased volatility when moving from discretion to commitment.
5 Counterfactual Analysis

Outcomes over the sample period are more consistent with the idea that the Fed set policy under discretion rather than commitment. Not only does the discretionary model fit better, inferences about the relative importance of interest rate smoothing accord with traditional views on the primary goals of monetary policy. These findings motivate the following question. Assuming that historical policy was indeed discretionary, how much better off would outcomes have been had the Fed committed to an optimal rule? This section performs counterfactual simulations to ascertain how the economy might have evolved under commitment.

Figure 5 displays the actual series for inflation, the output gap, and the interest rate and the paths these variables would have taken had the Fed implemented commitment from 1982:Q1 to 2008:Q4. To produce counterfactual data, the fixed interval Kalman smoother described by de Jong (1989) is used to estimate the “true” history of shocks implied by the discretionary model. The shocks are then reinserted into the model, holding the parameters fixed (including loss function weights) but with policy shifted to commitment. Preliminary simulations revealed that full commitment would not have been operational due to frequent violations of the zero lower bound on the nominal interest rate. To assess the gains from commitment while respecting the presence of the zero bound, I follow Woodford (2003a, Ch. 6) in approximating the effects of this constraint by amending the loss function to include a penalty on the variance of the interest rate. By placing a large enough weight on the auxiliary term, the probability that the nonnegativity constraint ever binds can be made arbitrarily small. In this exercise I selected a weight on interest rate smoothing just sufficient to ensure that the counterfactual series is nonnegative at every date over the sample period.

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19 The smoothed estimates of the shocks reflect information contained in the full sample.
20 In a related exercise Dennis (2005) estimates shocks from a model with a forward-looking Taylor rule. He then simulates two sets of counterfactual data by feeding the shocks back into the model but with the Taylor rule replaced by a calibrated loss function. One simulation assumes commitment and the other discretion.
21 A value of $\lambda_i = 0.0024$ in the loss function $E_0(1-\delta) \sum_{t=0}^{\infty} \delta^t \{ \pi_t^2 + \lambda_y y_t^2 + \lambda_\delta (i_t - \bar{i}_{t-1})^2 + \lambda_i i_t^2 \}$ guarantees a strictly positive counterfactual sequence of nominal interest rates.
Figure 5 shows that the interest rate path would have been different under commitment. Rates would have fallen more quickly in the 1980s, bottoming out at 3.80 percent in 1988:Q1 and staying near historical levels during the 1990s. After peaking at 7.43 percent in 2000:Q4, the interest rate would have trended down for the rest of the sample. Overall, the volatility of the simulated path is larger than what actually transpired. The biggest gap between the counterfactual and observed series is 3.76 percentage points, occurring in 2002:Q4.

Despite very different policy behavior at times, the output gap and inflation paths would have been strikingly similar to historical outcomes. Inflation would have been smaller before 1995:Q4 under commitment and slightly larger thereafter. The maximum gap between the two series is only 0.48 percentage points in 1990:Q3. The output gap also would have been slightly lower in the late 1980s and early 1990s but a bit higher starting in the mid 1990s.

Although useful for historical comparisons, counterfactual simulations do not easily translate into a single measure that quantifies the cumulative losses associated with one policy relative to another. For this purpose I follow Jensen (2002) and Dennis and Söderström (2006) and compute the “inflation equivalent,” interpreted as the permanent increase in inflation from target that in terms of central bank loss is equivalent to moving from commitment to discretion. The inflation equivalent can be calculated from (3) as \( \pi_{eq} = \sqrt{L_d - L_c} \), where \( L_d \) and \( L_c \) are the losses under discretion and commitment, respectively.\(^{22}\) A similar quantity measured in terms of lost output, an output gap equivalent, is given by \( y_{eq} = \pi_{eq} / \sqrt{\lambda_y} \).

Table 5 reports loss under both policies and the inflation and output gap equivalents from discretion. It also reports the variances of inflation, the output gap, and the interest rate in first differences and levels. Moving from discretion to commitment while respecting the zero bound leads to small reductions in output gap and inflation variability. It also lowers the variance of the interest rate in first differences but increases it in levels. Loss under

\(^{22}\)A permanent inflation rate of \( \pi_{eq} \) yields a loss equal to \( (1 - \delta) \sum_{j=0}^{\infty} \delta^j \pi_{eq}^2 = \pi_{eq}^2 \). Hence, the inflation equivalent satisfies \( L_c + \pi_{eq}^2 = L_d \).
discretion is about 6 percent higher than commitment, which is equivalent to a sustained rise in inflation of 0.37 percentage points or an output gap of 1.17 percentage points.

6 An Optimized Simple Rule

This section estimates a version of the model that specifies policy with a simple Taylor-type rule rather than commitment or discretion. Following McCallum (1999), I assume that the simple rule permits feedback from lagged endogenous variables only. Exogenous shocks and current endogenous variables are excluded on the grounds that neither would be observable to actual policymakers when setting the interest rate. The class of policy rules considered is

\[
i_t = \theta_i i_{t-1} + (1 - \theta_i)(\theta_\pi \pi_{t-1} + \theta_{y1} y_{t-1} + \theta_{y2} y_{t-2}),
\]

where \(\theta_\pi, \theta_{y1}, \theta_{y2}, \) and \(\theta_i\) measure the central bank’s response to the lagged state.

In keeping with the theme of optimal policy, the central bank chooses \(\Theta = \{\theta_\pi, \theta_{y1}, \theta_{y2}, \theta_i\}\) once and for all to minimize (3) subject to (1), (2), and (13). The role of policy thereafter is to implement the optimized rule in every period. When solving for \(\Theta\), it takes into account how the promise to carry out (13) at all future dates impacts private expectations. Clarida et al (1999) refer to this type of policy design as “commitment” to an optimal simple rule.\(^{23}\)

Before discussing the results, I address an issue concerning estimation of the standard errors. As noted in Salemi (2006), maximum-likelihood estimation of a model with an optimal simple rule can produce a likelihood function that is not differentiable near the extremum. Consequently, methods that rely on approximating the Hessian or the outer product of the score are incapable of generating meaningful standard errors. An inspection of the likelihood surface for the present model reveals similar irregularities. Thus, I adopt a Bayesian strategy to quantify the uncertainty regarding parameter estimates. The procedure involves coupling

\(^{23}\)The domain of \(\Theta\) is restricted to ensure that (13) yields a determinate rational expectations equilibrium.
the likelihood function with a prior distribution over the parameters using Bayes’ theorem to form a joint posterior distribution from which the sampling variability can be inferred. Following Onatski and Williams (2010), I formulate independent uniform prior densities for the 11 structural parameters. The bounded ranges over which the priors are defined permit a large area of the parameter space to be explored when constructing the posterior. To generate draws from the posterior, I employ the Metropolis-Hastings algorithm described in An and Schorfheide (2007). The draws are used to approximate the posterior mean and 95 percent confidence interval for each parameter.24 The findings are reported in Table 6.

Also in Table 6 are the posterior mode estimates, found by maximizing the sum of log likelihood and the log-prior distribution. It turns out that the posterior mode is identical to the estimate one would obtain by maximizing log likelihood over the support of the prior. The reason is because a uniform prior views all points within the support as equally probable and those outside as having zero probability, so it does not inform the data in any significant way. It merely truncates the range of values deemed permissable for maximum likelihood.25

The parameters in the IS equation are the ones most affected by the switch to an optimal simple rule. For example, demand shocks $\sigma_y$ roughly double when moving from commitment or discretion to a simple rule. This shift is statistically significant as estimates of $\sigma_y$ under the original policies lie outside the 95 percent probability interval. Estimates of $\sigma$ and $\phi$ also change considerably under a simple rule. The estimate of the former rises to 0.05 while the latter drops to 0.14. These results are similar to ones in Salemi (2006). Using an optimized simple rule identical to (13), Salemi estimates $\sigma$ and $\phi$ to be 0.042 and 0.162, respectively.

As for the loss function, the estimate of $\lambda_y = 0.10$ is almost unchanged from the discretion case. Draws from the posterior indicate a high degree of precision in this estimate, with the 95 percent confidence interval spanning 0.04 to 0.13. Regarding interest rate smoothing, the

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24 Details concerning posterior density estimation can be found in the working paper available upon request.

25 None of the mode estimates fall near the edges of the prior, so classical and Bayesian inference are the same in this case. Adding priors serves only to enable estimation of posterior means and confidence intervals.
estimate of $\lambda_{\Delta_1} = 0.76$ falls between the values obtained under commitment and discretion. The posterior distribution of $\lambda_{\Delta_1}$ is also concentrated tightly around the mean, suggesting that the data are highly informative about this parameter.

The central bank’s control problem defines $\Theta$ as an implicit function of the structural parameters and loss function weights. The optimized rule for the estimates in Table 6 is

$$i_t = 0.8923i_{t-1} + (1 - 0.8923)(3.0510\pi_{t-1} + 8.9252y_{t-1} - 7.9048y_{t-2}).$$

(14)

Reconciling an optimal simple rule with the data evidently requires large countercyclical responses to inflation and the output gap in the long run but only gradual adjustment of the interest rate to this desired level in the short run.

I conclude with a brief discussion of model fit. Among the three policies estimated in this paper, discretion produces the highest log-likelihood value followed by commitment and then the simple rule. Pseudo-odds ratios point to discretion as the dominant model, assigning near zero probability to the other two. The relative performance of the simple rule also highlights the implications for model fit of allowing policy to respond contemporaneously to economic shocks. A key difference between commitment or discretion and the policy rule (13) is that the latter restricts the information set to include only lagged state variables. When policy conditions on an expanded state that accommodates current demand and supply shocks, log likelihood improves by 8 points in the commitment case and 16 points in the discretion case.

#### 7 Concluding Remarks

This paper reports estimates from a forward-looking model of the US economy in which monetary policy minimizes the central bank’s loss function. The model is estimated separately under commitment and discretion using maximum likelihood over the Volcker-Greenspan-
Bernanke era. The goal is to judge which mode of optimal policy fits the data best and to see whether the two procedures generate statistically different parameter estimates. The results point to broad similarities in the estimates across policies with one major exception. The weight on interest rate smoothing in the loss function is large under commitment but small under discretion. This result can be traced to the fact that commitment increases interest rate volatility, and maximum likelihood tries to compensate by lifting the weight on policy smoothing. Measures of fit based on the likelihood function indicate that discretionary policy provides a superior description of the joint time-series properties of the data.

The foregoing empirical analysis uses a strictly binary framework in which policy choices are either full commitment or discretion. In future work it might be more realistic to think about Fed behavior as lying between these two extremes. Schaumburg and Tambalotti (2007) develop a modeling device, which they call “quasi-commitment,” that makes it possible to analyze a continuum of policies between commitment and discretion that differ in degree of credibility. Policymakers are understood by the public to renege on a commitment plan every period with some fixed probability. Outcomes converge to full commitment as this probability approaches zero and to discretion as it approaches one. Within a quasi-commitment framework, it should be possible to estimate simultaneously the weights in the central bank’s loss function and the exogenous probability that identifies its measure of credibility.

**Literature Cited**


### Table 1
Maximum-Likelihood Estimates

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Notes: The table reports maximum-likelihood estimates of the following model where $\ln L$ denotes the value of log likelihood:

$$y_t = \phi E_i y_{t+1} + (1 - \phi) (\beta y_{t-1}) + (1 - \beta) y_{t-2} - \sigma (i_t - E_t \pi_{t+1}) + u_{y,t},$$

$$\pi_t = \alpha E_i \pi_{t+1} + (1 - \alpha) \pi_{t-1} + \kappa y_t + u_{\pi,t},$$

$$L_t = E_t (1 - \delta) \sum_{j=0}^{\infty} \beta^j \{ \pi_{t+j}^2 + \lambda_y y_{t+j}^2 + \lambda_{\Delta i} (i_{t+j} - i_{t+j-1})^2 \}.$$
Table 2
Maximum-Likelihood Estimates of Restricted Models

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<td>0.5106</td>
<td>0.6250</td>
</tr>
<tr>
<td></td>
<td>(0.0068)</td>
<td>(0.0267)</td>
<td>(0.0076)</td>
<td>(0.0261)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.0049</td>
<td>0.0111</td>
<td>0.0053</td>
<td>0.0007</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td>(0.0055)</td>
<td>(0.0015)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$\ln L$</td>
<td>-389.8648</td>
<td>-383.6755</td>
<td>-396.5589</td>
<td>-379.6847</td>
</tr>
</tbody>
</table>

Notes: The table reports restricted maximum-likelihood estimates of the following model:

\begin{align*}
y_t &= \phi E_{t+1}y_{t+1} + (1 - \phi)(\beta y_{t-1} + (1 - \beta)y_{t-2}) - \sigma (i_t - E_{t+1}\pi_{t+1}) + u_{y,t}, \\
\pi_t &= \alpha E_{t+1}\pi_{t+1} + (1 - \alpha)\pi_{t-1} + \kappa y_t + u_{\pi,t}, \\
L_t &= E_t(1 - \delta) \sum_{j=0}^{\infty} \delta^j \left\{ \pi_{t+j}^2 + \lambda_y y_{t+j}^2 + \lambda_{\Delta i}(i_{t+j} - i_{t+j-1})^2 \right\}.
\end{align*}

$^a$ is a value that is set prior to estimation and $\ln L$ is log likelihood. Numbers in parentheses are standard errors.
## Table 3
### Standard Deviations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Commitment</th>
<th>Discretion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>1.0822</td>
<td>2.2140</td>
<td>1.2298</td>
</tr>
<tr>
<td>Output Gap</td>
<td>2.2506</td>
<td>4.5958</td>
<td>2.7194</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>2.4144</td>
<td>3.7923</td>
<td>3.0239</td>
</tr>
</tbody>
</table>

*Notes: Standard deviations are multiplied by 100.*
Table 4
Model Comparison

<table>
<thead>
<tr>
<th>Model</th>
<th>Log likelihood</th>
<th>$BIC$</th>
<th>Pseudo-odds $(z = 2)$</th>
<th>Pseudo-odds $(z = 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commitment</td>
<td>-387.7724</td>
<td>-413.4729</td>
<td>0.0003</td>
<td>0.0002</td>
</tr>
<tr>
<td>Discretion</td>
<td>-379.6072</td>
<td>-405.3078</td>
<td>0.9997</td>
<td>0.7481</td>
</tr>
<tr>
<td>Unrestricted</td>
<td>-371.3507</td>
<td>-406.3969</td>
<td>—</td>
<td>0.2517</td>
</tr>
</tbody>
</table>

Notes: $BIC$ is the Bayesian information criterion. The pseudo-odds statistic measures the data-determined probability of a model $j$ and is defined as $\rho(j) = \exp(BIC(j))/\sum_{h=1}^{z} \exp(BIC(h))$, where $z$ is the number of distinct models under consideration.
Table 5
Counterfactual Losses under Discretion

<table>
<thead>
<tr>
<th>Policy</th>
<th>Var((\pi))</th>
<th>Var((y))</th>
<th>Var((\Delta i))</th>
<th>Var((i))</th>
<th>Loss</th>
<th>(\pi^{eq})</th>
<th>(y^{eq})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretion</td>
<td>1.5124</td>
<td>7.3952</td>
<td>2.3420</td>
<td>9.1442</td>
<td>2.2148</td>
<td>0.3665</td>
<td>1.1665</td>
</tr>
<tr>
<td>Commitment</td>
<td>1.4045</td>
<td>7.2419</td>
<td>1.8775</td>
<td>14.5813</td>
<td>2.0805</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Notes: The table reports variances of inflation \(\pi\), the output gap \(y\), and the interest rate in differences \(\Delta i\) and levels \(i\) as well as loss under discretion and commitment. The loss differential is also reported as an inflation (output gap) equivalent \(\pi^{eq}\) (\(y^{eq}\)).
Table 6
Estimation Results for an Optimal Simple Rule

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Dist.</th>
<th>Post. Mean</th>
<th>95% Prob. Int.</th>
<th>Post. Mode</th>
<th>Commitment</th>
<th>Discretion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_y$</td>
<td>[0.10, 0.90]</td>
<td>0.4922</td>
<td>[0.39, 0.58]</td>
<td>0.4289</td>
<td>0.2482</td>
<td>0.2065</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>[0.10, 0.90]</td>
<td>0.5818</td>
<td>[0.51, 0.67]</td>
<td>0.5639</td>
<td>0.5745</td>
<td>0.6108</td>
</tr>
<tr>
<td>$\sigma_{\gamma\pi}$</td>
<td>[−0.20, 0.00]</td>
<td>−0.0723</td>
<td>[−0.12, −0.02]</td>
<td>−0.0675</td>
<td>−0.0381</td>
<td>−0.0352</td>
</tr>
<tr>
<td>$\lambda_y$</td>
<td>[0.50, 1.50]</td>
<td>1.0628</td>
<td>[0.92, 1.24]</td>
<td>1.0249</td>
<td>0.9274</td>
<td>0.9403</td>
</tr>
<tr>
<td>$\lambda_{\Delta_i}$</td>
<td>[0.25, 1.25]</td>
<td>0.7447</td>
<td>[0.70, 0.78]</td>
<td>0.7602</td>
<td>2.5581</td>
<td>0.0579</td>
</tr>
<tr>
<td>$\phi$</td>
<td>[0.01, 0.40]</td>
<td>0.0739</td>
<td>[0.01, 0.19]</td>
<td>0.1395</td>
<td>0.3286</td>
<td>0.3747</td>
</tr>
<tr>
<td>$\beta$</td>
<td>[1.30, 1.70]</td>
<td>1.5402</td>
<td>[1.39, 1.67]</td>
<td>1.5029</td>
<td>1.5153</td>
<td>1.4483</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>[0.01, 0.10]</td>
<td>0.0613</td>
<td>[0.04, 0.09]</td>
<td>0.0460</td>
<td>0.0089</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>[0.30, 0.70]</td>
<td>0.5178</td>
<td>[0.51, 0.53]</td>
<td>0.5109</td>
<td>0.4927</td>
<td>0.6162</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>[0.001, 0.010]</td>
<td>0.0023</td>
<td>[0.001, 0.004]</td>
<td>0.0016</td>
<td>0.0045</td>
<td>0.0047</td>
</tr>
</tbody>
</table>

| $\ln L$ | −395.6592 | −387.7724 | −379.6072 |
| $BIC$ | −421.3598 | −413.4729 | −405.3078 |
| $\rho$ | 0.0000 | 0.0003 | 0.9997 |

Notes: The table reports estimation results for the following model:

$$
y_t = \phi y_{t+1} + (1 - \phi)(\beta y_{t-1} + (1 - \beta)y_{t-2}) - \sigma (i_t - E_t \pi_{t+1}) + u_{y,t}, $$

$$
\pi_t = \alpha E_t \pi_{t+1} + (1 - \alpha) \pi_{t-1} + \kappa y_t + u_{\pi,t}, $$

$$
i_t = \theta_i y_{t-1} + \theta_{i1} y_{t-2} + \theta_{i2} y_{t-3} + \theta_{i3} y_{t-4},$$

$$
L_t = E_t (1 - \delta) \sum_{j=0}^{\infty} \beta^j \{ \pi_{t+j}^2 + \lambda \gamma y_{t+j}^2 + \lambda_{\Delta}(i_{t+j} - i_{t+j-1})^2 \},$$

where $\{ \theta_i, \theta_\pi, \theta_{i1}, \theta_{i2} \} = \arg \min \ln L_t$. The first column displays the support of the uniform prior for each parameter. The next two columns report the posterior means and 95 percent probability intervals generated from the Metropolis-Hastings algorithm described in An and Schorfheide (2007). The fourth column presents maximum-likelihood estimates, which are equal to the posterior mode estimates under independent uniform priors. The last two columns reproduce the estimates under full commitment and discretion, where numbers in parentheses are Hessian-based standard errors. $\ln L$ denotes log likelihood and $BIC$ is the Bayesian information criterion. The pseudo-odds statistic measures the data-determined probability of a model $j$ and is defined as $\rho(j) = \exp(BIC(j))/\sum_{h=1}^{z} \exp(BIC(h))$, where $z$ is the number of distinct models under consideration.
Fig. 1. The figure shows vector autocorrelation functions for the output gap $y$, inflation $\pi$, and the nominal interest rate $i$ implied by US data (solid line), the commitment model (dashed line), and the discretionary model (dotted line).
Fig. 2. The figure displays impulse responses of the output gap $y$, inflation $\pi$, and the nominal interest rate $i$ to a demand shock $u_{y,t}$ (left column) and a supply shock $u_{\pi,t}$ (right column). Response functions are graphed for the estimated model under discretion (solid line) and the commitment model using the point estimates obtained under discretion (dashed line). Each panel traces out the effect of a one-standard-deviation shock, and the values are interpreted as percent deviations from steady state.
Fig. 3. The figure displays standard deviations of inflation (solid line), the output gap (dashed line), and the nominal interest rate (dotted line) as the weight on interest-rate smoothing $\lambda_{\Delta i}$ varies from its estimated value, holding the other parameters fixed at their point estimates. The left panel graphs the functions implied by the estimated discretion model. The right panel corresponds to the commitment model. Vertical lines indicate the estimated values of $\lambda_{\Delta i}$ and crosses identify sample moments.
Fig. 4. The figure displays impulse responses of the output gap $y$, inflation $\pi$, and the nominal interest rate $i$ to a demand shock $u_{y,t}$ (left column) and a supply shock $u_{\pi,t}$ (right column). Response functions are graphed for the estimated discretionary model (solid line) and the estimated commitment model (dotted line). Each panel traces out the effect of a one-standard-deviation shock, and the values are interpreted as percent deviations from steady state.
Fig. 5. The figure plots the actual series as implied by the estimated discretionary model (solid lines) and the counterfactual series generated under commitment (dotted lines) with the zero lower-bound restriction.