14 Policy Evaluation with a Forward-Looking Model

Rouben V. Atoian, Gregory E. Givens and Michael K. Salemi[1]

> Monetary policy rules are naturally amenable to modern econometric policy evaluation methods that were developed as part of the rational expectations revolution in macroeconomics in the early 1970's. When using these methods, researchers first build a structural model of the economy, consisting of mathematical equations with estimated numerical parameter values. They then test out different rules by simulating the model stochastically.... One monetary policy rule is better than another...if the simulation results show better economic performance. (Taylor, 1998).

Modern econometric policy evaluation entails two steps. In the first, the parameters of a structural model are either estimated or obtained through calibration. In the second, the performance of alternative policy rules is studied and conclusions about policy are reached. Fuhrer (1997) is a good example of the two part approach. He fits a small structural model to data for the US economy treating the coefficients of his policy rule as free parameters. He then derives an optimal policy frontier by varying the values of the policy-rule coefficients to minimize a weighted sum of the variance of output and the variance of inflation. He evaluates policy by comparing the variances of output and inflation achieved by the estimated rule with points on the policy frontier.

In this paper, we follow standard practice by setting out a small structural model, obtaining estimates of its parameters, and then evaluating

[1]Salemi is the corresponding author and may be reached via email at Michael_Salemi@unc.edu. The authors thank colleagues at UNC for valuable comments. Remaining errors are our own. Michael Salemi thanks participants at the European Monetary Forum Conference on Money in honor of Sir Alan Walters for their helpful comments on a related paper.

the performance of alternative policy rules while treating estimates of the structural parameters as fixed and known. We break with standard practice in an interesting way. Our maintained hypotheses include an auxiliary assumption that permits us to identify the covariance matrix of structural errors. On the assumption that structural covariances as well as structural parameters are known and fixed, we are able to compare the performance of backward- and forward-looking fixed coefficient rules. We are also able to compare the performance of fixed-coefficient rules to the performance of the optimal commitment policy and to the optimal policy under discretion. Our paper thus provides evidence on the practical importance to a central bank of obtaining a commitment mechanism and the loss in performance when the commitment mechanism takes the form of a simple and verifiable fixed-coefficient policy rule.

We evaluate the performance of a policy with a loss function with three inputs. The first input is a set of three weights that represent the relative importance to the central bank of stabilizing inflation, output, and interest rates. We compute optimal policies and corresponding loss values for different policy weights in order to determine whether conclusions about the relative performance of policies are sensitive to policy objectives. Policy weights range between zero and one and sum to one. The second input to the loss function is the covariance matrix of structural errors. For fixed-coefficient rules, we use the Klein algorithm to compute the covariance matrix of reduced form errors from the structural error covariance matrix, the structural parameters, and the coefficients of the policy rule. The reduced form error covariance matrix is then used to compute policy loss. The third input to the loss function is the state-transition coefficient matrix. For fixed-coefficient rules, we compute this matrix with the Klein algorithm. For optimal commitment and discretion, we derive the reduced form and compute policy loss with a version of Soderlind's (1999) algorithm.

Our policy analysis supports several interesting findings. First, the original Taylor rule, with *a priori* coefficient values, performs quite well when stabilizing inflation and stabilizing output are both important objectives. Its performance can, however, be very poor for other sets of weights. Second, for a wide variety of policy-objective weights, backward-looking rules perform as well as or better than rules that permit the central bank to adjust the rate of interest in response to current output and inflation. In fact, a backward looking rule which permits the central bank to condition the rate of interest on the full state vector for the economy is the best performer for more than half of our policy objective weight configurations. Third, when the central bank makes output stabilization its chief objective, an optimized version of the Taylor rule

where the interest rate depends on current values of output and inflation and the lagged interest rate is the best performer among the rules we consider.

We begin in section 1 with an example that explains how the coefficients of a policy rule are computed when the structure is " backward-looking." The example highlights the challenges associated with computing optimal policies for " forward-looking" models. Section 1 also describes the algorithm we use to compute policy loss. In section 2, we set out the forward-looking structural model that underlies our analysis and explain how we use the Klein algorithm to solve it and compute policy loss. In section 3, we present the results of our policy evaluation for fixed coefficient rules. In section 4, we explain how we compute loss for optimal commitment and discretionary policies and compare results for these policies with results for fixed-coefficient policies described in section 3. Our concluding remarks are contained in section 5.

1 Optimal Policy with a Backward Looking Model.

We begin with an example where monetary policy is like a game against nature in the sense that the parameters of the economy's state transition equation are independent of the policy chosen by the central bank. If the state transition equation is linear and the bank's objective function is quadratic, optimal policy is characterized by the matrix Ricatti equations. Given regularity conditions, backward iteration of the Ricatti equations shows that optimal policy is a fixed-coefficient rule. The example permits us to highlight the challenges that arise when, in contrast, the structural equations of the model are forward looking and optimal policies and state transition equations must be simultaneously determined.

The example is built around a three equation model for output, inflation, and the interest rate. The central bank wishes to stabilize the time paths of output and inflation by controlling the interest rate (r). Stabilizing output means keeping it close to its long run growth path. Stabilizing the inflation rate means keeping it constant. To keep the notation simple, y and p are defined as differences of output and inflation from target values so that the central bank wants to keep y and p as close to zero as possible.

The model is composed of three structural equations.

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} - b\,(r_t - p_t) + u_t \tag{1}$$

$$p_t = \beta y_t + \alpha p_{t-1} + v_t \tag{2}$$

$$r_t = \theta_1 y_{t-1} + \theta_2 p_{t-1} + \theta_3 r_{t-1} + \theta_4 y_{t-2} + w_t \tag{3}$$

Equation (1) is a backward looking IS schedule which implies that equilibrium output is inversely related to the real rate of interest which, for now, is defined as the interest rate minus the current inflation rate. Equation (2) is a backward looking Phillips curve which implies that inflation tends to rise when output exceeds its steady state value. The lagged values of y in equation (1) and p in equation (2) capture the effects of partial adjustment mechanisms and govern the dynamic responses of output and inflation to shocks. Equation (3) explains how the central bank adjusts the nominal interest rate in response to changes in the economy. A monetary policy is a set of values for the parameters of the feedback equation. Structural shocks (u, v, and w) are assumed to have zero means and to be serially uncorrelated.

The model restricts monetary policy in two ways. First, the interest rate is a function only of past values of output and inflation which implies that the central bank can not respond contemporaneously to demand and supply shocks. Because the state of the economy is completely described by y_{t-1}, p_{t-1}, r_{t-1}, and y_{t-2}, adding additional lagged variables to the right hand side of (3) is superfluous. Second, the values of θ_1 through θ_4 are fixed, a sufficient but not a necessary condition for a time-consistent policy.

For equations (1)–(3), monetary policy is a game against nature because the parameters of the state transition equation for output and inflation are constant and independent of monetary policy. The reduced form y and p may be written as

$$Z_t = AZ_{t-1} + Cr_t + U_t \tag{4}$$

where $Zt = (y_t, p_t, r_t, y_{t-1})'$, $U = (\eta_{1t}, \eta_{2t}, 0, 0)'$, $\eta_{1t} = d(u_t + bv_t)$, $\eta_{2t} = d(\beta u_t + v_t)$, $d = (1 - b\beta)^{-1}$ and where A and C are matrices given by:

$$A = \begin{bmatrix} da_1 & db\alpha & 0 & da_2 \\ d\beta\alpha_1 & d\alpha & 0 & dba_2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -db \\ -db\beta \\ 1 \\ 0 \end{bmatrix}$$

We assume that the central bank chooses values for θ_1 through θ_4 that minimize the loss function

$$\Lambda = E_0 \sum_{t=0}^{\infty} \delta^t Z_t' W Z_t \tag{5}$$

where W is a (4×4) matrix of policy weights that determine the relative importance to the central bank of its stabilization objectives and where δ is the central bank's time rate of discount. We assume W is diagonal

with $W_{1,1} = Wy$, $W_{2,2} = Wp$, $W_{3,3} = Wr$, $W_{4,4} = 0$ where Wy, Wp, and Wr are the weights assigned by the central bank to stabilizing output, inflation, and the interest rate. Since what matters is the relative size of weights, we normalize the sum of the weights to 1.0.

Because the state transition equation is linear and its objective function is quadratic, the central bank is a linear regulator and the solution to its problem is given by:

$$r_t = \Theta Z_{t-1} + w_t \tag{6}$$

where $\Theta = (\theta_1, \theta_2, \theta_3, \theta_4)$ is the (1×4) vector of reaction function coefficients[2]. The optimal value for Θ is the limit to the series Θ_T, Θ_{T-1}, Θ_{T-2}, computed with the Matrix Ricatti equations:

$$\begin{aligned}
H_T &= \delta^T W \\
\Theta_T &= -\left(C' H_T C\right)^{-1} C' H_T A \\
H_{T-1} &= \delta^{T-1} W + A' H_T \left(A + C\Theta_T\right) \\
\Theta_{T-1} &= -\left(C' H_{T-1} C\right)^{-1} C' H_{T-1} A \\
&\vdots \\
H_{T-j} &= \delta^{T-j} W + A' H_{T-j-1} \left(A + C\Theta_{T-j-1}\right)
\end{aligned} \tag{7}$$

Certainty equivalence holds. The solution to the central bank problem is the same as the solution to the companion problem where random shocks are absent from the structural equations (Sargent, 1987). Inspection of the Ricatti equations confirms that the optimal reaction function coefficients do not depend on the covariance matrix of the model's error terms. McGratten (1990) reports that it is computationally efficient to compute the optimal θ by iterating the Ricatti equations to convergence.

For the forward looking model presented in the following section, the optimal reaction function coefficients are not characterized by the Ricatti equations and must be computed by numerical minimization of loss. To see how this can be done, write the reduced form for y, p and r as a first-order vector autoregression:

$$X_t = GX_{t-1} + \Phi_t \tag{8}$$

where $X_t = (y_t, p_t, r_t, y_{t-1}, p_{t-1}, r_{t-1})'$, $\Phi_t = (\varphi_1, \varphi_2, \varphi_3, 0, 0, 0)'$, $\varphi_{1t} = d(u_t + bv_t - bw_t)$, $\varphi_{2t} = d(\beta u_t + v_t - b\beta w_t)$, $\varphi_{3t} = w_t$ and the (6×6)

[2]Strictly speaking there should not be an error present the reaction function. Hansen and Sargent (1980) explain how to account for an error in a policy rule. An adaptation of the Hansen-Sargent argument to the current setting is given by Salemi (1995, p. 421).

matrix G is

$$G = \begin{bmatrix} G_{11} & G_{12} \\ 1 & 0 \end{bmatrix}$$

$$G_{11} = \begin{bmatrix} d(a_1 - b\theta_1) & db(\alpha - \theta_2) & -db\theta_3 \\ d\beta(a_1 - b\theta_1) & d(\alpha - b\beta\theta_2) & -db\beta\theta_3 \\ \theta_1 & \theta_2 & \theta_3 \end{bmatrix}$$

$$G_{12} = \begin{bmatrix} d(a_2 - b\theta_4) & 0 & 0 \\ d\beta(a_2 - b\theta_4) & 0 & 0 \\ \theta_4 & 0 & 0 \end{bmatrix}$$

Because they are linear combinations of the serially uncorrelated structural errors, the φ_{jt} are serially uncorrelated. The moving average representation for X_t is $(I - GL)^{-1}\Phi_t$ where L is the lag operator.

Next write Λ as a function of the forecast error variances of the model's variables.

$$\begin{aligned}
\Lambda &= E_0 \sum_{t=0}^{\infty} \delta^t X_t^{'} \tilde{W} X_t \\
&= \sum_{t=0}^{\infty} \delta^t \text{ trace} \left[\tilde{W} E_0 (X_t X_t')\right] \\
&= \text{trace} \left[\tilde{W} \sum_{t=0}^{\infty} \delta^t E_0 (X_t X_t')\right] \\
&= \text{trace} \left[\tilde{W} \sum_{t=0}^{\infty} \delta^t \left(E_0 (X_t - E_0 X_t)(X_t - E_0 X_t)' + (E_0 X_t)(E_0 X_t)'\right)\right] \\
&= \text{trace} \left[\tilde{W}(M + N)\right] \qquad (9)
\end{aligned}$$

where $\tilde{W}$ is a (6×6) diagonal matrix with $(1,1)$, $(2,2)$, and $(3,3)$ element equal to W_y, W_p, and W_r and with zeroes elsewhere. Λ involves two sums:

$$M = \sum_{t=0}^{\infty} \delta^t E_0 (X_t - E_0 X_t)(X_t - E_0 X_t)'$$

and

$$N = \sum_{t=0}^{\infty} \delta^t (E_0 X_t)(E_0 X_t)'$$

M is the discounted sum of forecast error variances of X computed at time zero when policy is set. N is the discounted sum of quadratic

terms in expected departures of X from its target. Provided that the economy is on target at the time when policy is set, $N = 0$ and the objective of the central bank is to minimize the part of Λ that involves M. If the economy begins away from its target path, the central bank faces a tradeoff between returning the economy to its target path and minimizing the weighted sum of discounted error variances. Throughout this paper we assume $N = 0$.

The last step is derivation of a convenient expression for M. Let Ω be the (6×6) covariance matrix for Φ_t with $\Omega_{1,1}$, the (3×3) covariance matrix for the non-zero elements of Φ, in the upper left corner and zeroes elsewhere. Because Φ_t is serially uncorrelated, we have

$$E_0 (X_t - E_0 X_t) (X_t - E_0 X_t)' = \Omega + G\Omega G' + G^2 \Omega \left(G^2\right)' + \ldots + G^{k-1} \Omega \left(G^{k-1}\right)' \quad (10)$$

and

$$M = \Omega + \delta [\Omega + G\Omega G'] + \ldots + \delta^k \left[\Omega + G\Omega G' + \ldots + G^{k-1} \Omega \left(G^{k-1}\right)'\right] + \ldots = (1 - \delta)^{-1} \left[\Omega + \delta G \Omega G' + \delta^2 G^2 \Omega \left(G^2\right)' + \ldots\right] \quad (11)$$

The direct minimization strategy computes M by iterating the square-bracket term in (11) to convergence and computes loss as trace $(\tilde{W} M)$. Alternative techniques for computing M are discussed in Anderson *et al* (1996)[3].

2 Optimal Policy with a Forward Looking Model.

In this section, we discuss computation of optimal policies for a forward-looking structural model in which agents have rational beliefs about future values of output and inflation.

$$y_t = \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b (r_t - E_t p_{t+1}) + u_t \quad (12)$$

$$p_t = \beta y_t + \alpha_1 E_t p_{t+1} + \alpha_2 p_{t-1} + v_t \quad (13)$$

$$r_t = \theta_1 y_{t-1} + \theta_2 p_{t-1} + \theta_3 r_{t-1} + \theta_4 y_{t-2} + w_t \quad (14)$$

The IS schedule (12) may be obtained by combining a linearized Euler equation that characterizes a representative household's optimal choice between consumption and saving and the market clearing condition for output. As explained by Clarida, Gali, and Gertler (1999),

[3]The Matlab programs we used to compute optimal reaction function coefficients are available on request.

the presence of expected future output in the IS equation results from the desire of households to smooth consumption. When households expect higher consumption in the future, they want to consume more in the present which raises the current level of aggregate demand and, in equilibrium, introduces a positive association between the current and expected future levels of output. The presence of lagged output in the IS equation can be explained by habit persistence or adjustment costs. Woodford (1996) and Bernanke, Gertler, and Gilchrist (1998) provide the details. Svensson (2000) adapts the story to an open economy.

If α_2 is zero, equation (13) is a version of the new Phillips curve discussed by Gali and Gertler (1999), Clarida, Gali, and Gertler (1999), and Svensson (2000). The foundation for the new Phillips curve is a model in which monopolistically competitive firms adjust their prices on a staggered basis as in Calvo (1983). When it has the opportunity, an individual firm adjusts its price to maximize expected profits while taking account of the restriction it faces on future price adjustment and the expected future prices of its competitors. The staggered-price-setting story leads to an equation where the current rate of inflation is a function of the firm's current level of marginal cost and the expected future inflation rate. The new Phillips curve results when the output gap (y) is used as a proxy for marginal cost.

If α_2 is not zero, equation (13) is a version of the new hybrid Phillips curve developed by Gali and Gertler to explain inertia in the rate of inflation. The foundation is a model with two kinds of firms. The first kind is a Calvo firm. The second kind is a follower that sets its current price equal to the average of prices set by competitors in the previous period plus an adjustment for inflation. The existence of backward looking firms is sufficient to introduce lagged inflation into the Phillips curve. Alternatively, Clarida, Gali, and Gertler (1999) account for lagged inflation in the Phillips curve by assuming serially correlated supply shocks.

As before, the model includes a fixed-coefficient reaction function (14) and the central bank chooses coefficient values to minimize expected loss. Equation (14) is essentially the same as (9) of Fuhrer and Moore (1995) and (4) of Fuhrer (1997).

Equations (12)–(14) introduce two layers of complexity to the control problem of the central bank. First, because agents' actions depend upon expected future output and inflation, there may be zero or many reduced form equations for y_t, p_t and r_t. Second, because agents' beliefs are rational, changes in Θ cause changes in the parameters of the state transition equation. Thus, Θ and the state transition equation must be solved for simultaneously.

We address the issues of solution existence and multiplicity using

the extension of Blanchard and Kahn (1980) proposed by Klein (2000). Equations (12)–(14) are written in Klein format as

$$\tilde{A}\begin{bmatrix} Z_t \\ E_t y_{t+1} \\ E_t p_{t+1} \end{bmatrix} = \tilde{B}\begin{bmatrix} Z_{t-1} \\ y_t \\ p_t \end{bmatrix} + \tilde{C}s_t \tag{15}$$

where $Z_t = (y_t, p_t, r_t, y_{t-1})'$, $S_t = (u_t, v_t, w_t)'$, and where $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ are given by:

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -b & 0 & \lambda & b \\ 0 & 0 & 0 & 0 & 0 & \alpha_1 \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \theta_1 & \theta_2 & \theta_3 & \theta_4 & 0 & 0 \\ -a_1 & 0 & 0 & -a_2 & 1 & 0 \\ 0 & -\alpha_2 & 0 & 0 & -\beta & 1 \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

In the language of Klein, Z_{t-1} is the vector of backward-looking variables and y_t and p_t are the forward-looking variables. The Klein solution strategy computes a generalized "QZ" decomposition of $\tilde{A}$ and $\tilde{B}$. For any pair of conformable square matrices $\left(\tilde{A}, \tilde{B}\right)$, there exist orthonormal matrices Q and Z and upper triangular matrices S and T such that

$$\tilde{A} = Q'SZ' \qquad \tilde{B} = Q'TZ \qquad QQ' = ZZ' = I$$

The generalized eigenvalues of the system are the ratios T_{ii}/S_{ii} where T_{ii} and S_{ii} are the diagonal elements of T and S. Without loss of generality, the decomposition matrices can be transformed so that the generalized eigenvalues are arrayed in ascending modulus order (Klein, 2000, p. 1410).

Provided that the number of stable eigenvalues equals the number of backward-looking variables, Theorem 5.1 in Klein shows that the unique

solution for the backward looking variables is given by

$$Z_t = \left(Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1}\right) Z_{t-1} + L S_t \tag{16}$$

where Z_{11}, S_{11}, and T_{11}, are the (4×4) upper left blocks of Z, S, and T and where L is a (4×3) matrix given by (5.23) in Klein. For our model, a unique solution will exist if there are four stable and two unstable eigenvalues.

Given our assumption that the Fed reacts neither to current values of output and inflation nor to current structural shocks, we can recover Σ, the covariance matrix of structural errors, from Ω, the covariance matrix of reduced form errors with the mapping $\Sigma = L^{-1}\Omega\left(L'\right)^{-1}$. We exploit this mapping in the policy experiments described in the following section.

With the forward-looking model, the central bank control problem is complicated by the fact that the parameters of the state transition equation depend on Θ. What the central bank may take as fixed is the structure of the economy and not its reduced form. An algorithm that the central bank could use to compute the coefficients of its policy rule has three steps. First, the algorithm chooses a starting value for Θ, uses (16) to compute the reduced form and the resulting G matrix, and then computes policy loss using (9) and (11). Second, it calculates partial derivatives of loss with respect to each element of Θ. For every change in Θ, G must be re-computed because private agents respond to policy changes by changing their beliefs and actions. Third, the algorithm updates Θ when doing so lowers policy loss provided that the Klein saddle path restriction is satisfied. The algorithm repeats steps two and three until it can no longer lower policy loss.

3 Monetary Policy Rules

A monetary policy rule specifies how a central bank will respond to changes in economic conditions. If the coefficients of the rule are chosen optimally, the rule is also an explicit commitment to a set of policy objectives. But why should a central bank adopt commitment in the form of a fixed coefficient rule?

The case for commitment builds on the realization that policy effectiveness depends not only on policy actions but also on public understanding of those actions and public expectations of future actions (Kydland and Prescott, 1977). Policy is more effective when its future course is predictable. Lacking a commitment mechanism, the central bank has an incentive to exploit stickiness in wages and prices to damp

recessions. The public reacts with inflation expectations that incorporate future discretionary stimulus.

Commitment permits the central bank to distribute "policy medicine" over time. For example, suppose the central bank wishes to offset inflation that will result from a supply shock. Under commitment, it can raise interest rates moderately provided that it maintains higher rates for a period of time. Lacking commitment, a higher initial rate increase will be necessary because the public doubts that the central bank will sustain the rate increase.

Optimal commitment need not take the form of a fixed-coefficient reaction function. It is a state-contingent plan that gives the instrument setting as a function of the history of exogenous shocks. Optimal commitment is not practical for two reasons. First, it is not feasible to provide an advance listing of all relevant contingencies (Woodford, 2002). Second, it is difficult for the public to distinguish between discretion and a complicated contingency rule. Both problems are avoided when the central bank commits to a fixed-coefficient rule

What form should a fixed-coefficient rule take? Most industrialized-economy central banks use a short-term interest rate as their control variable. An obvious example is the US Federal Reserve which sets a target level for the federal funds rate and controls the supply of bank reserves to keep the funds rate at the target. Because the Fed is able to closely control the federal funds rate, it makes sense to treat the funds rate itself as the policy instrument. In what follows, we limit attention to fixed coefficient rules that explain how the short-term interest rate should be adjusted in response to economic conditions.

The most famous examples of interest rate rules are those proposed by John Taylor which in our notation may be written as:

$$r_t = \theta_p p_t + \theta_y y_t + \theta_r r_{t-1} \tag{17}$$

The original Taylor rule (Taylor, 1993) assigns coefficient values that Taylor describes as providing both a sensible rule and an accurate description of Federal Reserve policy: $\theta_p = 1.5$, $\theta_y = 0.5$, and $\theta_r = 0$. The intuition for the large value of θ_p is that the central bank must raise the interest rate by more than any increase in inflation in order to raise the real rate of interest, cool the economy, and move inflation back toward its target. An interesting alternative to the original Taylor rule is a rule that sets θ_r to zero but chooses the values for θ_p and θ_r that minimize the loss function of the central bank. Taylor (1999) suggests another alternative that allows for interest-rate smoothing so that θ_r is positive. Mc Callum (1997) and others argue that policymakers can react only to lagged and not to current values of output and inflation. In response, Taylor (1999) suggests an alternative where lagged values of output and

inflation replace the current values in (17). In what follows, we will study the performance of all four forms of the Taylor rule.

The second type of rule we consider is the "full state" rule given by equation (14). There is one important difference between this rule and the lagged Taylor rule. Given our model, equation (14) permits the central bank to respond to all, rather than a subset, of the variables in the state vector. In theory, equation (14) would permit the central bank to better respond to business cycle momentum by conditioning the interest rate both on y_{t-1} and y_{t-2}. In practice, it is not clear whether conditioning policy on the full state vector will appreciably improve the performance of the rule. By comparing the performance of (14) and the Taylor rules, we can gather evidence on how important it is for the central bank to correctly specify the state vector.

Woodford (2002) attributes to Goodhart a simple rule where the central bank responds only to departures of the inflation rate from its target value. In terms of (17), the Goodhart rule amounts to setting $\theta_y = \theta_r = 0$ and choosing an optimal value for θ_p. Batini and Haldane (1998) recommend rules where the central bank reacts to expected future inflation. Clarida, Gali, and Gertler (1998) also suggest that forecast-based rules are optimal for a central bank with a quadratic objective function such as ours. We implement these recommendations with a version of (17), called the expected inflation rule, where $\theta_y = \theta_r = 0$, $E_t[p_{t+1}]$ replaces p_t, and where θ_p is chosen to minimize policy loss.

Slope Parameters						
λ	a_1	a_2	b	α_1	α_2	β
0.230	1.06	−0.305	0.030	0.600	0.401	.0006
Error Covariances ($\times 10^{-5}$)						
σ_{uu}	σ_{vv}	σ_{ww}	σ_{uv}	σ_{uw}	σ_{vw}	
1.53	1.38	1.78	−0.172	0.610	0.342	

Table 1: Structural Parameter Values Used to Compare Policy Rules

Our policy evaluation is based on estimates of the coefficients of (12)–(14) obtained by Salemi (2002) and reported in Table 1. Salemi fits (12)–(14) to quarterly data for the U.S. for 1983–2001 subject to the restriction that the coefficients of the policy rule minimize a quadratic loss function. We assume that correlation between the error (w_t) in the policy rule and the errors in the IS schedule (u_t) and the Phillips curve (v_t) are the result of contemporaneous responses of output and inflation to w_t rather than to the contemporaneous response of policy to structural shocks. It is then straightforward to back out an estimate of the structural error covariance matrix from Salemi's estimate of the reduced

form error covariance matrix. Our estimate of the structural error covariance matrix is also reported in Table 1. We follow the literature by treating our estimates of structural parameters as fixed and known values. In future work, we intend to extend our analysis by treating the parameters as random variables.

Our results are summarized in Table 2 and in Figures 1–6. Table 2 reports the policy rule that achieved the lowest loss level for each set of policy-objective weights considered. The Table takes the form of a triangular grid with W_p, the inflation weight, across the columns and W_y, the output weight, along the rows[4]. Nodes on the diagonal represent cases in which minimal weight was assigned to stabilizing the rate of interest. Nodes above the diagonal represent cases where higher weight was assigned to the objective of interest rate smoothing.

Table 2: The Minimum Loss Fixed Coefficient Rule

Legend
E: Expected Inflation Rule
F: Full-State Rule
T: Taylor Rule with Interest Rate Smoothing

Wy\Wp	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
0	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
0.05	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	F	F	
0.10	E	E	E	E	E	E	E	E	E	E	E	E	F	F	F	F		
0.15	E	E	E	E	E	E	E	E	F	F	F	F	F	F	F			
0.20	E	E	E	E	F	F	F	F	F	F	F	F	F	F				
0.25	F	F	F	F	F	F	F	F	F	F	F	F	F					
0.30	F	F	F	F	F	F	F	F	F	F	F	F						
0.35	F	F	F	F	F	F	F	F	F	F	F							
0.40	F	F	F	F	F	F	F	F	F	T								
0.45	F	F	F	F	F	F	F	F	T									
0.50	F	F	F	F	F	F	F	T	T									
0.55	F	F	F	F	F	F	T	T										
0.60	F	F	F	F	T	T	T											
0.65	F	F	T	T	T	T												
0.70	T	T	T	T	T													
0.75	T	T	T	T														
0.80	T	T	T															
0.85	T	T																
0.90	T																	

Our first finding is that the best policy rule is always one of three: the single-coefficient expected inflation rule (EI), the full state rule (FS), and the version of the Taylor rule in which the rate of interest is conditioned on current output, current inflation, and the lagged rate of interest and

[4]In some cases when we allowed $W_p = 1$ or $W_r = 0$, our loss-minimization algorithm did not converge. For this reason, we restricted attention to values of $W_r \geq 0.05$ and values of $W_p \geq 0.05$.

the coefficients are chosen to minimize loss (TS). EI is the best rule either when zero weight is assigned to stabilizing output or when very-substantial weight is assigned to interest rate smoothing. TS is the best rule when $W_y \geq 0.70$ no matter the distribution of weight across the other objectives. FS is the best rule when $0.25 \leq W_y \leq 0.35$ no matter the distribution of weights across other objectives. For other values of W_y, the optimal rule can be any of the three depending on the weight assigned to the other two objectives.

There are two interesting implications of our best rule findings. First, a simple rule in which the rate of interest is made a function only of the expected rate of inflation can be the best fixed-coefficient policy rule but only in the case where the central bank cares nothing about stabilizing output or greatly dislikes variability in the rate of interest. When even modest weight is assigned to output stability, FS produces lower loss than EI. Second, the advantage conferred upon the TS rule of conditioning the rate of interest on current rather than past values of output and inflation is valuable only when output stabilization is the dominant objective. In most nodes along the diagonal of the Table, where interest rate stability is given little weight, FS performs better. If the Federal Reserve considers inflation stabilization to be its primary objective and output stabilization to be an important but secondary objective, it would be well advised to adopt an interest rate rule of the form of (14).

Figures 1–6 provide quantitative evidence on the relative performance of the rules. Figure 1 is a graph of the ratio of policy loss for the original Taylor rule to the policy loss for the full state rule. Given that Taylor assigned values to the coefficients on a priori grounds that did not include minimizing policy loss, it is not surprising that the ratio always exceeds one. What is surprising is how poor the relative performance of the original Taylor rule can be. Taylor-rule loss is much higher when W_y is small and when W_r is near zero. However, it is very interesting that the original Taylor rule performs almost as well as the full-state rule when $W_p = 0.80$ and $W_y = W_r = 0.10$ which in our view is not a bad guess about the preferences of the Federal Reserve since the end of the monetarist experiment.

Figure 2 plots the ratio of policy loss for the optimized Taylor rule to policy loss for the full state rule. Since this Taylor rule conditions the interest rate on current values of output and inflation, Figure 2 provides a referendum on the value of conditioning policy on current rather than lagged economic variables. The figure shows that the optimized Taylor rule performs slightly better when W_r is very small and when $W_y \geq 0.70$. On the other hand, the optimized Taylor rule performs relatively poorly

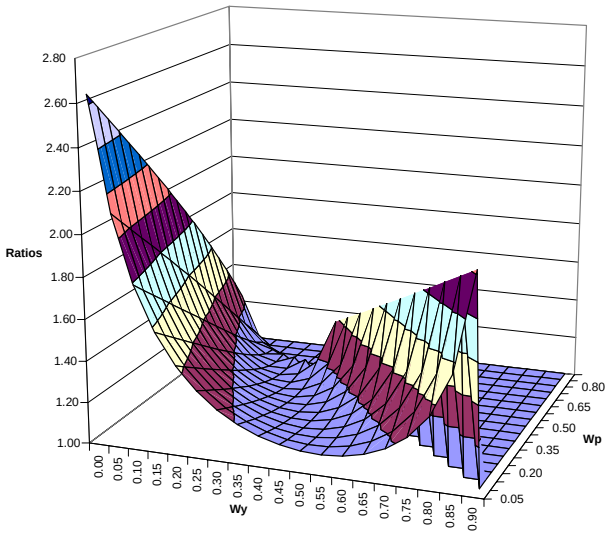

Figure 1: Policy Loss Ratio: Original Taylor Rule/Full-State Rule

when interest rate smoothing is important and when W_p is large. This later finding is not surprising since the optimized Taylor rule does not allow the interest rate to be conditioned on its lagged value.

Figure 3 shows compares the performance of the full state rule to that of the version of the Taylor rule that allows for interest rate smoothing. Again, the coefficients of the Taylor rule are those that minimize loss. The most striking thing about the figure is that except for extreme values of W_y the performance of the two rules is quite close. This Taylor rule continues to have an advantage when W_r is very small; the full state rule has an advantage when $W_r \geq 0.10$. It appears that conditioning policy on current values of output and inflation involves a tradeoff between the benefits of more current information and the costs of a more volatile interest rate. Figure 4 compares the original Taylor rule to the Taylor rule with coefficients on output and inflation chosen to minimize loss. It confirms one of the conclusions supported by Figure 1. The optimized Taylor rule always performs better, but the performance of the two rules is nearly the same when W_p is large and W_y and W_r are of modest size.

Figure 5 compares the Goodhart rule with the full state rule. The Goodhart rule is the simplest interest rate rule we consider since it adjusts the nominal rate of interest only in response to departures of the current inflation rate from target values. The figure shows that the

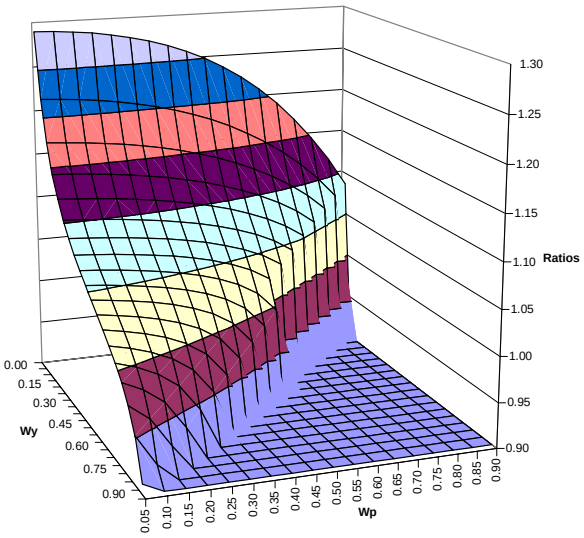

Figure 2: Policy Loss Ratio: Optimized Taylor Rule/Full-State Rule

Goodhart rule never performs better than the full state rule despite its information advantage. The relative performance of the Goodhart rule is better when W_y is very small and, particularly, when W_p is very large. However, the full state rule performs much better for large values of W_y and for large values of W_p combined with modest values of W_y. We conclude that the central bank of an economy well described by our model ought not adopt a Goodhart rule. Figure 6 tells a similar story about the performance of the rule in which the nominal rate of interest responds only to changes in the current expectation of future inflation. The backward looking full state rule outperforms this forward looking rule unless a very high weight is placed on the interest rate stabilization objective. The full state rule performs better when W_p is sizeable even if W_y is very small. A central bank that cares mostly about stabilizing inflation and is not too concerned about interest rate stability would do better adopting the full state rule.

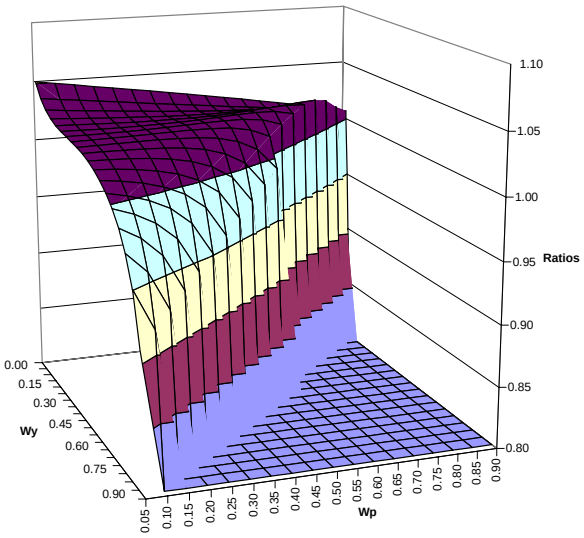

Figure 3: Policy Loss Ratio: Taylor-Smoothing Rule/Full State Rule

4 Optimal Commitment and Discretion

In the previous section, we evaluated the economic performance of a set of fixed-coefficient policy rules. In this section, we compare the performance of our rules to that of two alternatives which Clarida, Gali, and Gertler (1999) call "unconstrained optimal commitment policy" and "discretionary policy."

The unconstrained optimal commitment (commitment) policy is fundamentally different from fixed-coefficient rules. Rules "live" in the space spanned by the current state vector for an economic model. The commitment policy depends on the entire history of the state vector dating back to time zero when policy is set. At time zero, the central bank evaluates all possible outcomes, decides how to react to each, and promises to stick with the chosen set of reactions.

To explain how we compute the commitment policy, we modify our notation to conform to that of Soderlind (1999) and write the constraint facing the central bank as:

$$\tilde{A}\begin{bmatrix} Z_{t+1} \\ E_t y_{t+1} \\ E_t p_{t+1} \end{bmatrix} = \tilde{B}\begin{bmatrix} Z_t \\ y_t \\ p_t \end{bmatrix} + \tilde{C} r_t \begin{bmatrix} S_{t+1} \\ 0 \end{bmatrix} \tag{18}$$

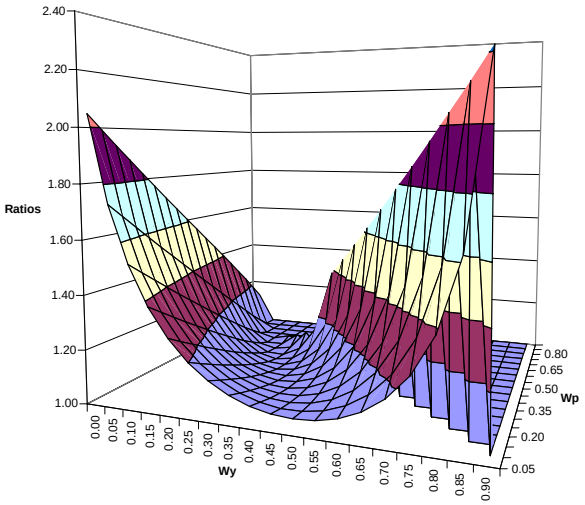

Figure 4: Policy Loss Ratio: Original Taylor Rule/Optimized Taylor Rule

where Z_t is redefined to include the structural errors from the IS equation and Phillips curve so that $Z_t = (u_t, v_t, y_{t-1}, p_{t-1}, r_{t-1}, y_{t-2})'$, where $S_t = (u_t, v_t)'$, and where the elements of $\tilde{A}, \tilde{B}$ and $\tilde{C}$ are obtained in a straightforward way by re-writing the structural equations in the above format. There are two essential differences between (15) and (18). First, the structural shocks are now considered to be part of the state vector permitting the interest rate under commitment to depend on current and past values of those shocks. Second, the interest rate is assumed to exactly equal the value specified by the commitment policy so that w_t , the interest rate shock, is assumed to be zero.

To characterize the commitment policy, we adopt the approach of Currie and Levine (1993) and formulate the LaGrangian function:

$$J_0 = E_0 \sum_{t=0}^{\infty} \delta^t \left[\begin{array}{c} W_p (p_t)^2 + W_y (y_t)^2 + W_r (r_t)^2 \\ +2\lambda_{t+1} \left(\tilde{B} X_t + \tilde{C} r_t + \varepsilon_t - \tilde{A} X_{t+1} \right) \end{array} \right] \tag{19}$$

where $X_t = (Z_t', E_t(y_{t+1}), E_t(p_{t+1}))'$ and where, $\varepsilon_t = (S_t, 0)'$. We compute the commitment policy by using Klein's method to solve simultaneously a system of equations comprising (18) and the first-order conditions for the optimization problem. To compute the value of the loss function associated with the optimal policy we apply equation (4.15) of

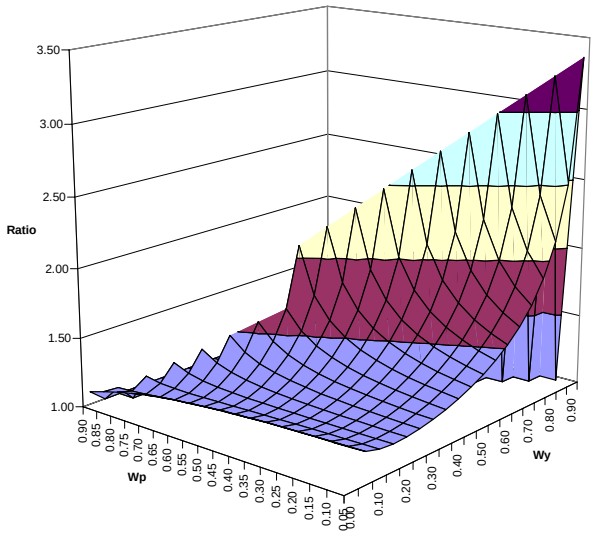

Figure 5: Policy Loss Ratio: Goodhart Rule/Full-State Rule

Ljungqvist and Sargent (2000)[5].

The second alternative policy design we consider is optimal discretion. What distinguishes discretion from commitment is that current and past policy decisions in no way constrain future decisions. Under discretion, the central bank re-optimizes its loss function (5) every period taking private sector expectations as exogenous. Under commitment, the central bank optimizes only in the inaugural period and treats private agent expectations as endogenous and changing with policy. Under commitment, the central bank simultaneously chooses paths for the interest rate and private sector expectations subject to the constraints imposed by the economic structure. Under discretion, the central bank lacks credibility and has no control over private agents expectations. An alert private sector adjusts expectations according to actual policy decisions. In the context of our model, private agents predict central bank decisions by solving the central bank loss minimization problem while recognizing that the bank is free to change policy. The outcome of the "game" played by the central bank and private agents is an equilibrium for which the central bank has no incentive to change policy although it

[5]For the detailed description of the algorithms used in these computations, see Givens (2002).

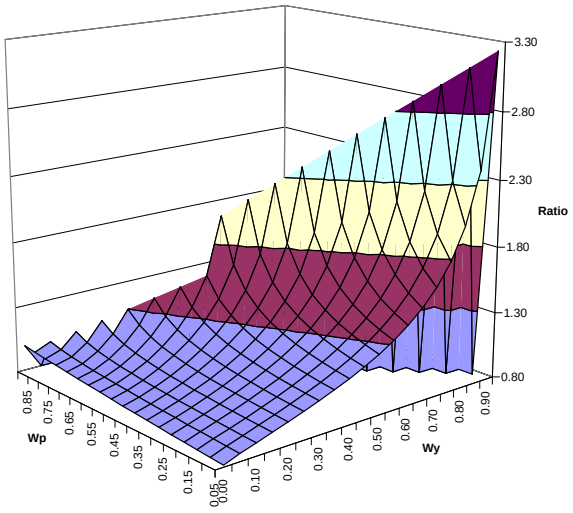

Figure 6: Policy Loss Ratio: Expected Inflation Rule/Full-State Rule

has the ability to do so.

To characterize discretion, we adopt the same notation used for commitment and formulate the following Bellman equation.

$$\Lambda_t = Z_t' V_t Z_t + d_t = W_p \left(p_t\right)^2 + W_y \left(y_t\right)^2 + W_r \left(r_t\right)^2 + \delta E_t \left(Z_{t+1}' V_{t+1} Z_{t+1} + d_{t+1}\right) \quad (20)$$

The solution under discretion involves minimizing (20) over choice of r_t, where V_t is a 6×6 positive definite, symmetric matrix and d_t is a scalar. Both values are initially undetermined and are found by solving for the fixed point of a particular system of equations. The equations we use are those explained in detail in Soderlind (1999)[6]. The optimal policy is a fixed coefficient feedback rule that relates the nominal interest rate to the current state of the economy. Unlike the commitment policy, the rule under discretion will depend only on the current state vector and not on its entire history.

Figure 7 reports the ratio of policy loss for discretion relative to optimal commitment. To compute loss, we use the parameter values from

[6]For the details, see Givens (2002).

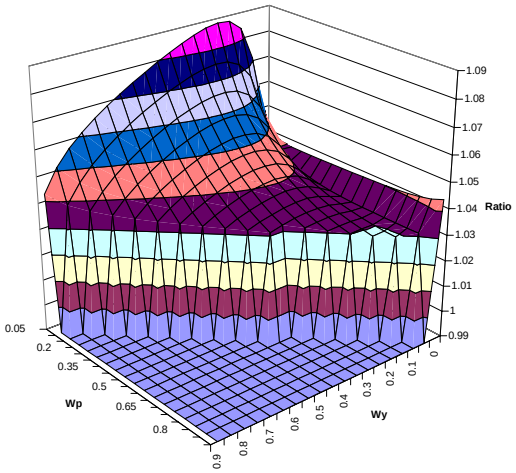

Figure 7: Policy Loss Ratio: Discretion/Commitment

Table 1, assume that variance of the error from the interest rate equation is zero, and then implement the programs described earlier in this section. Several conclusions are warranted. First, loss computed under discretion always exceeds loss computed under commitment. This is no surprise. Second, provided that W_r is very small, the loss ratio is about the same for all values of W_p. (Recall that $W_p + W_y + W_r = 1.0$) We find this result surprising — we expected that commitment would do relatively better when inflation stabilization was the more important objective. Third, the relative performance of discretion worsens as more weight is placed on interest rate stability. This occurs because the interest rate is more volatile under discretion than under commitment. Fourth, for a given W_p, the relative loss ratio first rises and then falls with increases in W_y. For a given value of W_y , the loss ratio increases monotonically as W_r rises and W_p falls.

Figures 8 and 9 compare loss under optimal commitment and loss under discretion with loss under the full state fixed coefficient rule described in section 3. We use the full state rule for comparison because it was the lowest-loss rule for a wide variety of policy objectives. In order to make a valid comparison across these three policy designs, we re-computed optimal full state coefficients and loss values under the assumption that σ_w^2, the variance of the error in the interest rate equation, is zero.

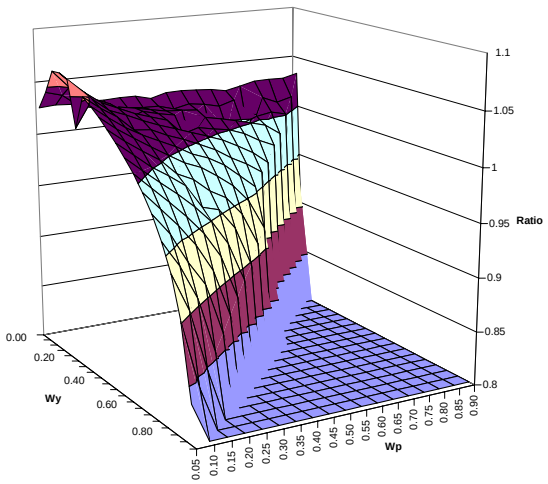

Figure 8: Policy Loss Ratio: Discretion/Full-State Rule

Figure 8 shows that loss associated with the full state rule exceeds loss associated with discretion whenever W_r is small so that interest rate stabilization is relatively unimportant. As W_r increases, the relative performance of the full state rule improves. The full state rule produces lower loss than discretion when $W_r > 0.35$. This is consistent with our earlier finding that the ratio of loss under discretion to loss under commitment is largest when interest rate stability is relatively important. Our finding that discretion can outperform the full state rule should be viewed in context. Discretion outperforms the full state rule for a subset of policy weights quite similar to the subset for which the optimized Taylor rule outperforms the full state rule.

Figure 9 confirms that loss associated with the full state rule always exceeds loss associated with optimal commitment. The full state rule falls furthest short of the commitment potential when W_r is small. For $W_r = 0.05$, the ratio of loss under commitment to loss under the full state rule is about 0.80 when W_y is 0.90 and fall steadily as W_p increases. The ratio is 0.99 when W_p is .90. Thus, as inflation stabilization becomes a more important objective, the full state rule, despite conditioning the rate of interest on lagged values of output and inflation, very nearly achieves the full commitment potential.

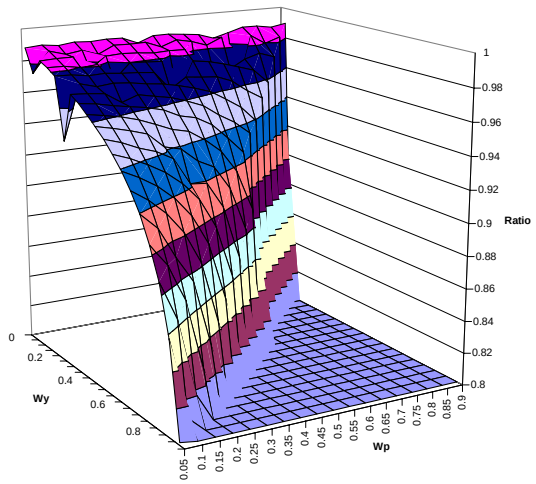

Figure 9: Policy Loss Ratio: Commitment/Full-State Rule

5 Concluding Remarks

We conclude by repeating our key findings. First, computation of optimal feedback parameters of a fixed-coefficient policy rule requires the researcher to account for the effects of changes in those coefficients on private agent expectations and reduced form parameters. We accomplish this complicated task with a Matlab program that marries Klein's solution algorithm with an iterative strategy for solving a Sylvester equation. Second, of the half dozen fixed coefficient rules we studied, one of three always performs best. The rule where the interest rate responds only to the current expectation of future inflation performs best when inflation and interest rate stability are the sole objectives of policy. We find it remarkable that a single-parameter rule could ever outperform all the other rules we consider. As output stabilization becomes a more important objective, one of two rules dominates. The full state rule, where the interest rate varies with lagged values of output and inflation, is the best rule for about half of the weight configurations that we consider, especially for those where $0.25 \leq W_y \leq 0.45$. The version of the Taylor rule that allows for interest rate smoothing and that has coefficients chosen to minimize loss is the best rule whenever $W_y \geq 0.70$. Third, the difference between policy loss under optimal commitment and policy loss under discretion ranges between three and nine percent, with the great-

est disparity observed when interest rate stability is relatively important. Fourth, discretion can result in lower loss than commitment with a fixed coefficient policy rule. Fifth, when inflation stability is the dominant objective of the central bank, loss under the full state rule is nearly as small as loss under optimal commitment and substantially lower than loss under discretion. When inflation stabilization is the primary objective of the central bank, fixed coefficient rules can nearly achieve the lowest loss possible.

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