Appendix to “On the Gains from Monetary Policy Commitment under Deep Habits”

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A. Symmetric Equilibrium Conditions

$$h_t^{\chi} x_t^{\sigma} = w_t(1-\tau) \tag{A.1}$$

$$a_t x_t^{-\sigma} = \beta R_t E_t \left(a_{t+1} x_{t+1}^{-\sigma} / \pi_{t+1} \right) \tag{A.2}$$

$$x_t = c_t - b c_{t-1} \tag{A.3}$$

$$w_t = mc_t z_t \tag{A.4}$$

$$\nu_t = 1 - mc_t + \beta b E_t \left(a_{t+1}/a_t \right) \left(x_{t+1}/x_t \right)^{-\sigma} \nu_{t+1} \tag{A.5}$$

$$c_t = \eta \nu_t x_t + \alpha(\pi_t/\pi - 1)(\pi_t/\pi) y_t - \alpha \beta E_t \left(a_{t+1}/a_t \right) \left(x_{t+1}/x_t \right)^{-\sigma} (\pi_{t+1}/\pi - 1)(\pi_{t+1}/\pi) y_{t+1} \tag{A.6}$$

$$y_t = z_t h_t \tag{A.7}$$

$$y_t = c_t + (\alpha/2)(\pi_t/\pi - 1)^2 y_t \tag{A.8}$$

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t} \tag{A.9}$$

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_{z,t} \tag{A.10}$$

A symmetric competitive equilibrium is a set of processes $\{c_t, x_t, h_t, w_t, mc_t, \nu_t, y_t, \pi_t\}_{t=0}^{\infty}$ that satisfies (A.1)-(A.8), given a sequence of nominal interest rates $\{R_t\}_{t=0}^{\infty}$, initial conditions c_{-1}, and exogenous stochastic processes $\{a_t, z_t\}_{t=0}^{\infty}$ that satisfy (A.9)–(A.10).

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B. Quadratic Welfare Criterion

The welfare criterion given by equation (10) in the manuscript corresponds to a second-order Taylor series approximation of the representative agent's expected lifetime utility. In equilibrium, expected lifetime utility can be written as

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[\frac{x_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\chi}}{1+\chi} \right].$$

A quadratic expansion of the first term in the infinite sum yields

$$a_t \frac{x_t^{1-\sigma}}{1-\sigma} = x^{1-\sigma} \left\{ \hat{x}_t + \hat{a}_t \hat{x}_t + \frac{1}{2}(1-\sigma)\hat{x}_t^2 \right\} + t.i.p. + \mathcal{O}(\|\varepsilon\|^3), \tag{B.1}$$

where $\|\varepsilon\|$ is a bound on the amplitude of the exogenous shocks, $\mathcal{O}(\|\varepsilon\|^3)$ are terms in the expansion that are of third order or higher, and *t.i.p.* collects terms that are independent of monetary policy. In deriving (B.1), I have made use of the following result:

$$\frac{X_t - X}{X} = \hat{X}_t + \frac{1}{2}\hat{X}_t^2 + \mathcal{O}(\|\varepsilon\|^3),$$

where for any variable X_t, $\hat{X}_t \equiv \log(X_t) - \log(X)$ and X is the steady-state value of X_t.

The equilibrium equation for habit-adjusted consumption, $x_t = c_t - bc_{t-1}$, approximated to a second order is

$$\hat{x}_t = \frac{1}{1-b}\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) - \frac{b}{1-b}\left(\hat{c}_{t-1} + \frac{1}{2}\hat{c}_{t-1}^2\right) - \frac{1}{2}\hat{x}_t^2 + \mathcal{O}(\|\varepsilon\|^3).$$

Substituting this into (B.1) gives

$$a_t \frac{x_t^{1-\sigma}}{1-\sigma} = x^{1-\sigma} \left\{ \frac{1}{1-b}\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) - \frac{b}{1-b}\left(\hat{c}_{t-1} + \frac{1}{2}\hat{c}_{t-1}^2\right) + \hat{a}_t \hat{x}_t - \frac{1}{2}\sigma \hat{x}_t^2 \right\} + t.i.p. + \mathcal{O}(\|\varepsilon\|^3). \tag{B.2}$$

The discounted sum of (B.2) computed over all future periods can be written as

$$\sum_{t=0}^{\infty} \beta^t a_t \frac{x_t^{1-\sigma}}{1-\sigma} = x^{1-\sigma} \sum_{t=o}^{\infty} \beta^t \left\{ \frac{1-\beta b}{1-b}\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) - \frac{1}{2}\sigma \hat{x}_t^2 + \hat{a}_t \hat{x}_t \right\} + t.i.p. + \mathcal{O}(\|\varepsilon\|^3).$$

Here I have used the result that

$$\sum_{t=0}^{\infty}\beta^t\left(\hat{c}_{t-1}+\frac{1}{2}\hat{c}_{t-1}^2\right)=\left(\hat{c}_{-1}+\frac{1}{2}\hat{c}_{-1}^2\right)+\beta\sum_{t=0}^{\infty}\beta^t\left(\hat{c}_t+\frac{1}{2}\hat{c}_t^2\right).$$

The consumption terms dated before period zero are treated as initial conditions and are therefore independent of monetary policy.

A quadratic expansion of the second term in the agent's lifetime utility function gives

$$a_t\frac{h^{1+\chi}}{1+\chi}=h^{1+\chi}\left\{\hat{h}_t+\frac{1}{2}(1+\chi)\hat{h}_t^2+\hat{a}_t\hat{h}_t\right\}+t.i.p.+\mathcal{O}(\|\varepsilon\|^3). \tag{B.3}$$

Now in equilibrium, the production function becomes $y_t = z_t h_t$, which yields the exact relationship $\hat{y}_t = \hat{z}_t + \hat{h}_t$. Substituting this expression into (B.3) gives

$$a_t\frac{h^{1+\chi}}{1+\chi}=h^{1+\chi}\left\{\hat{y}_t+\frac{1}{2}(1+\chi)\hat{y}_t^2-(1+\chi)\hat{y}_t\hat{z}_t+\hat{a}_t\hat{y}_t\right\}+t.i.p.+\mathcal{O}(\|\varepsilon\|^3). \tag{B.4}$$

Discounting and summing (B.4) over all future periods then gives the following

$$\sum_{t=0}^{\infty}\beta^t a_t\frac{h_t^{1+\chi}}{1+\chi}=h^{1+\chi}\sum_{t=0}^{\infty}\beta^t\left\{\hat{y}_t+\frac{1}{2}(1+\chi)\hat{y}_t^2-(1+\chi)\hat{y}_t\hat{z}_t+\hat{a}_t\hat{y}_t\right\}+t.i.p.+\mathcal{O}(\|\varepsilon\|^3).$$

In order to express the welfare criterion in terms of output rather than consumption, it is helpful to consider the aggregate resource constraint $y_t = c_t + (\alpha/2)(\pi_t/\pi - 1)^2 y_t$. A second-order approximation of this expression yields

$$\hat{c}_t+\frac{1}{2}\hat{c}_t^2=\hat{y}_t+\frac{1}{2}\hat{y}_t^2-\frac{\alpha}{2}\hat{\pi}_t^2+\mathcal{O}(\|\varepsilon\|^3). \tag{B.5}$$

Expected lifetime utility can then be written as

$$\begin{aligned}V_0=x^{1-\sigma}E_0\sum_{t=o}^{\infty}\beta^t&\left\{\frac{1-\beta b}{1-b}\left(\hat{y}_t+\frac{1}{2}\hat{y}_t^2-\frac{\alpha}{2}\hat{\pi}_t^2\right)-\frac{1}{2}\sigma\hat{x}_t^2+\hat{a}_t\hat{x}_t\right\}\\&-h^{1+\chi}E_0\sum_{t=0}^{\infty}\beta^t\left\{\hat{y}_t+\frac{1}{2}(1+\chi)\hat{y}_t^2-(1+\chi)\hat{y}_t\hat{z}_t+\hat{a}_t\hat{y}_t\right\}+t.i.p.+\mathcal{O}(\|\varepsilon\|^3). \end{aligned}\tag{B.6}$$

As explained in section 3 of the manuscript, the labor-income tax rate τ appearing in the household's period budget constraint is calibrated so that there are no net distortions from market power or consumption externalities in the steady state. The tax rate that produces this result is given by $\tau = 1 - (1/mc)(1 - \beta b)$. Under these conditions, one can show that the steady-state consumption and labor allocations are Pareto efficient and satisfy $((1-\beta b)/(1-b))x^{1-\sigma} = h^{1+\chi}$. It follows that (B.6) simplifies to

$$V_0 = -\frac{1}{2}h^{1+\chi}E_0\sum_{t=o}^{\infty}\beta^t\left\{\alpha\hat{\pi}_t^2 + \frac{\sigma(1-b)}{1-\beta b}\hat{x}_t^2 + \chi\hat{y}_t^2 - 2(1+\chi)\hat{y}_t\hat{z}_t\right.$$
$$\left. + 2\hat{a}_t\hat{y}_t - 2\frac{1-b}{1-\beta b}\hat{a}_t\hat{x}_t\right\} + t.i.p. + \mathcal{O}(\|\varepsilon\|^3). \quad \text{(B.7)}$$

The next step is to transform the second-order terms in $\hat{y}_t$ and $\hat{x}_t$ into terms involving the "gap" variables, namely, the output gap $\hat{y}_t - \hat{y}_t^e$ and the habit-adjusted gap $\hat{x}_t - \hat{x}_t^e$. The variables y_t^e and x_t^e are the Pareto efficient levels of output and habit-adjusted output, respectively. These correspond to allocations that solve the benevolent planner's problem.

In solving for the Pareto efficient allocations, the planner maximizes lifetime utility subject to aggregate technology and feasibility constraints alone. The existence of price adjustment costs and market power do not constrain the planner's decisions. Moreover, all consumption habits are internalized in the course of optimization. In such an environment, efficiency requires that the marginal rate of substitution between work and habit-adjusted consumption equal the marginal product of labor, that is

$$a_t\left(\frac{y_t^e}{z_t}\right)^{\chi} = \left[a_t x_t^{e^{-\sigma}} - \beta b E_t a_{t+1}x_{t+1}^{e^{-\sigma}}\right]z_t. \quad \text{(B.8)}$$

The first-order approximation of (B.8) can be written as

$$\frac{\sigma}{1-\beta b}\left(\hat{x}_t^e - \beta b E_t\hat{x}_{t+1}^e\right) = \hat{z}_t + \frac{\beta b}{1-\beta b}\left(\hat{a}_t - E_t\hat{a}_{t+1}\right) - \chi\left(\hat{y}_t^e - \hat{z}_t\right). \quad \text{(B.9)}$$

Multiplying both sides of (B.9) by $-2\hat{y}_t$ yields

$$\chi\hat{y}_t^2 - 2(1+\chi)\hat{y}_t\hat{z}_t = \chi\left(\hat{y}_t - \hat{y}_t^e\right)^2 - 2\frac{\sigma}{1-\beta b}\hat{y}_t\left(\hat{x}_t^e - \beta b E_t\hat{x}_{t+1}^e\right) - \chi\hat{y}_t^{e^2}$$
$$+ 2\frac{\beta b}{1-\beta b}\hat{y}_t\left(\hat{a}_t - E_t\hat{a}_{t+1}\right), \quad \text{(B.10)}$$

where I have used the fact that $-2\chi\hat{y}_t\hat{y}_t^e = \chi\left(\hat{y}_t - \hat{y}_t^e\right)^2 - \chi\hat{y}_t^2 - \chi\hat{y}_t^{e^2}$.

Finally, note that $\hat{x}_t^2$ can be written as

$$\hat{x}_t^2 = \left(\hat{x}_t - \hat{x}_t^e\right)^2 + 2\hat{x}_t\hat{x}_t^e - \hat{x}_t^{e^2}. \quad \text{(B.11)}$$

Substituting (B.10) and (B.11) into (B.7) leads to the following expression for welfare:

$$
\begin{aligned}
V_0 = -\frac{1}{2}h^{1+\chi}E_0\sum_{t=o}^{\infty}\beta^t \Big\{ & \alpha\hat{\pi}_t^2 + \frac{\sigma(1-b)}{1-\beta b}\left(\hat{x}_t - \hat{x}_t^e\right)^2 + 2\frac{\sigma(1-b)}{1-\beta b}\hat{x}_t\hat{x}_t^e + \chi\left(\hat{y}_t - \hat{y}_t^e\right)^2 \\
& - 2\frac{\sigma}{1-\beta b}\hat{y}_t\left(\hat{x}_t^e - \beta b E_t\hat{x}_{t+1}^e\right) + 2\frac{\beta b}{1-\beta b}\hat{y}_t\left(\hat{a}_t - E_t\hat{a}_{t+1}\right) \\
& + 2\hat{a}_t\hat{y}_t - 2\frac{1-b}{1-\beta b}\hat{a}_t\hat{x}_t\Big\} + t.i.p. + \mathcal{O}(\|\varepsilon\|^3). \qquad \text{(B.12)}
\end{aligned}
$$

Equation (B.12) can be simplified further after recognizing that

$$
2\frac{\beta b}{1-\beta b}\hat{y}_t\left(\hat{a}_t - E_t\hat{a}_{t+1}\right) + \ 2\hat{a}_t\hat{y}_t - 2\frac{1-b}{1-\beta b}\hat{a}_t\hat{x}_t = 2\frac{b}{1-\beta b}\hat{y}_{t-1}\hat{a}_t - 2\frac{\beta b}{1-\beta b}\hat{y}_t E_t\hat{a}_{t+1},
$$

where I have used the fact that $\hat{x}_t = (1/(1-b))\left(\hat{y}_t - b\hat{y}_{t-1}\right)$. Discounting and summing the right-hand-side over all future periods and taking conditional expectations yields

$$
E_0\sum_{t=o}^{\infty}\beta^t\left\{2\frac{b}{1-\beta b}\hat{y}_{t-1}\hat{a}_t - 2\frac{\beta b}{1-\beta b}\hat{y}_t E_t\hat{a}_{t+1}\right\} = 2\frac{b}{1-\beta b}\hat{y}_{-1}\hat{a}_0 = t.i.p. \qquad \text{(B.13)}
$$

Substituting (B.13) into (B.12) then gives

$$
\begin{aligned}
V_0 = -\frac{1}{2}h^{1+\chi}E_0\sum_{t=o}^{\infty}\beta^t \Big\{ & \alpha\hat{\pi}_t^2 + \frac{\sigma(1-b)}{1-\beta b}\left(\hat{x}_t - \hat{x}_t^e\right)^2 + 2\frac{\sigma(1-b)}{1-\beta b}\hat{x}_t\hat{x}_t^e + \chi\left(\hat{y}_t - \hat{y}_t^e\right)^2 \\
& - 2\frac{\sigma}{1-\beta b}\hat{y}_t\left(\hat{x}_t^e - \beta b E_t\hat{x}_{t+1}^e\right)\Big\} + t.i.p. + \mathcal{O}(\|\varepsilon\|^3). \qquad \text{(B.14)}
\end{aligned}
$$

Finally, one can show that

$$
\frac{\sigma}{1-\beta b}E_0\sum_{t=o}^{\infty}\beta^t\hat{y}_t\left(\hat{x}_t^e - \beta b E_t\hat{x}_{t+1}^e\right) = \frac{\sigma b}{1-\beta b}\hat{y}_{-1}\hat{x}_0^e \ + \ \frac{\sigma(1-b)}{1-\beta b}E_0\sum_{t=o}^{\infty}\beta^t\hat{x}_t\hat{x}_t^e. \qquad \text{(B.15)}
$$

Using (B.15) to cancel terms in (B.14) leads to the following expression for welfare:

$$
V_0 = -\frac{1}{2}h^{1+\chi}E_0\sum_{t=o}^{\infty}\beta^t\left\{\alpha\hat{\pi}_t^2 + \chi\left(\hat{y}_t - \hat{y}_t^e\right)^2 + \frac{\sigma(1-b)}{1-\beta b}\left(\hat{x}_t - \hat{x}_t^e\right)^2\right\} + t.i.p. + \mathcal{O}(\|\varepsilon\|^3). \qquad \text{(B.16)}
$$

The quadratic approximation given by (B.16) is identical to the welfare criterion (10) displayed in section 3 of the manuscript.

C. Inputs for Computing Quasi-Commitment

Here I illustrate how to map the structural equations of the linearized deep habits model into companion form. The resulting matrix definitions can then be used to find a numerical solution to the government's quasi-commitment problem.

For sake of clarity and completeness, I begin by restating the equations:

$$\hat{x}_t = E_t\hat{x}_{t+1} - (1/\sigma)[\hat{R}_t - E_t\hat{\pi}_{t+1} - (1-\rho_a)\hat{a}_t], \tag{C.1}$$

$$\hat{x}_t = (1/(1-b))\hat{y}_t - (b/(1-b))\hat{y}_{t-1}, \tag{C.2}$$

$$\hat{mc}_t = \sigma\hat{x}_t + \chi\hat{y}_t - (1+\chi)\hat{z}_t, \tag{C.3}$$

$$\hat{\nu}_t = \beta b[E_t\hat{\nu}_{t+1} - (\hat{R}_t - E_t\hat{\pi}_{t+1})] - [\eta(1-b) - (1-\beta b)]\hat{mc}_t, \tag{C.4}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + (1/\alpha)[\hat{y}_t - \hat{\nu}_t - \hat{x}_t], \tag{C.5}$$

$$\hat{x}^e_t = \beta b E_t\hat{x}^e_{t+1} - (1/\sigma)[(1-\beta b)(\chi\hat{y}^e_t - (1+\chi)\hat{z}_t) - \beta b(1-\rho_a)\hat{a}_t], \tag{C.6}$$

$$\hat{x}^e_t = (1/(1-b))\hat{y}^e_t - (b/(1-b))\hat{y}^e_{t-1}, \tag{C.7}$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{a,t}, \tag{C.8}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + \varepsilon_{z,t}. \tag{C.9}$$

Define $\mathbf{x}_t = [\hat{a}_t\ \hat{z}_t\ \hat{y}_{t-1}\ \hat{y}^e_{t-1}]'$ the (4×1) vector of date-t predetermined variables, $\mathbf{X}_t = [\hat{x}_t\ \hat{y}_t\ \hat{mc}_t\ \hat{\nu}_t\ \hat{\pi}_t\ \hat{x}^e_t\ \hat{y}^e_t]'$ the (7×1) vector of date-t non-predetermined variables, $\boldsymbol{\varepsilon}_t = [\varepsilon_{a,t}\ \varepsilon_{z,t}]'$ the (2×1) vector of i.i.d. exogenous shocks, and $\mathbf{i}_t = \hat{R}_t$ the policy instrument. Stacking (C.1)–(C.9) in companion form produces the vector difference equation

$$\begin{bmatrix} \mathbf{x}_{t+1} \\ GE_t\mathbf{X}_{t+1} \end{bmatrix} = A\begin{bmatrix} \mathbf{x}_t \\ \mathbf{X}_t \end{bmatrix} + B\mathbf{i}_t + \begin{bmatrix} N\boldsymbol{\varepsilon}_{t+1} \\ 0 \end{bmatrix},$$

where matrices A and B are given by

$$A = \begin{bmatrix} \rho_a e_1 \\ \rho_z e_2 \\ e_6 \\ e_{11} \\ e_5 - ((1-\rho_a)/\sigma)e_1 \\ -e_5 + (1/(1-b))e_6 - (b/(1-b))e_3 \\ -e_7 + \sigma e_5 + \chi e_6 - (1+\chi)e_2 \\ -\sigma\beta b e_5 + e_8 + (\eta(1-b) - (1-\beta b))e_7 + \beta b(1-\rho_a)e_1 \\ e_9 - (1/\alpha)(e_6 - e_8 - e_5) \\ -e_{10} + (1/(1-b))e_{11} - (b/(1-b))e_4 \\ \kappa_1 e_{11} - b e_4 - \kappa_2 e_2 - (\beta b(1-b)(1-\rho_a)/\sigma)e_1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1/\sigma \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

and where $\kappa_1 \equiv 1 + \beta b^2 + \chi(1-\beta b)(1-b)/\sigma$ and $\kappa_2 \equiv (1-\beta b)(1-b)(1+\chi)/\sigma$. Similarly, matrices N and G are given by

$$N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad G = \begin{bmatrix} 1 & 0 & 0 & 0 & (1/\sigma) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sigma\beta b & 0 & 0 & \beta b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta b \end{bmatrix}.$$

In constructing A, I use the notation e_j, $j = 0, 1, ..., 11$, which denotes a 1×11 row vector with element j equal to one and all other elements equal to zero (for $j = 0$, $e_j = \mathbf{0}_{1\times 11}$).

Recall from section 5 of the manuscript that the approximate welfare criterion (10) or (B.16) can be written as $V_0 \approx -(1/2)h^{1+\chi}E_0 \sum_{t=0}^{\infty} \beta^t L_t$, where

$$L_t = \begin{bmatrix} \mathbf{x}_t \\ \mathbf{X}_t \\ \mathbf{i}_t \end{bmatrix}' W \begin{bmatrix} \mathbf{x}_t \\ \mathbf{X}_t \\ \mathbf{i}_t \end{bmatrix} = \alpha\hat{\pi}_t^2 + \chi(\hat{y}_t - \hat{y}_t^e)^2 + \frac{\sigma(1-b)}{1-\beta b}(\hat{x}_t - \hat{x}_t^e)^2. \tag{C.10}$$

In equation (C.10), W is a symmetric, positive semidefinite matrix whose elements contain the weights attached to the inflation and output gap stabilization objectives. Specifically, W is given by

$$W = \begin{bmatrix} e_0 \\ e_0 \\ e_0 \\ e_0 \\ (\sigma(1-b)/(1-\beta b))(e_5 - e_{10}) \\ \chi(e_6 - e_{11}) \\ e_0 \\ e_0 \\ \alpha e_9 \\ (\sigma(1-b)/(1-\beta b))(e_{10} - e_5) \\ \chi(e_{11} - e_6) \\ e_0 \end{bmatrix},$$

where this time e_j (for $j = 0, 1, ..., 12$) denotes a 1×12 row vector with element j equal to one and all other elements equal to zero (for $j = 0$, $e_j = \mathbf{0}_{1\times 12}$).

D. A Model with Aggregate Consumption Habits

D.1. Households

There is a unit measure of households, indexed by j, that gain utility from consuming a composite of differentiated goods $c_{j,t}$ and lose utility from supplying labor $h_{j,t}$. The composite good takes the form

$$c_{j,t} = \left[\int_0^1 c_{j,t}(i)^{1-1/\eta} di\right]^{1/(1-1/\eta)},$$

where $c_{j,t}(i)$ is consumption of good i by household j. The parameter $\eta > 1$ determines the intratemporal substitution elasticity across consumption varieties.

Every period household j minimizes $\int_0^1 P_t(i) c_{j,t}(i) di$ subject to the above aggregation constraint. First-order conditions imply demand functions of the form

$$c_{j,t}(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta} c_{j,t},$$

where $P_t(i)$ is the price of good i and $P_t \equiv \left[\int_0^1 P_t(i)^{1-\eta} di\right]^{1/(1-\eta)}$ is the price of $c_{j,t}$.

Intertemporal spending decisions are made with reference to a lifetime utility function

$$V_{j,0} = E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[\frac{(c_{j,t} - bc_{t-1})^{1-\sigma}}{1-\sigma} - \frac{h_{j,t}^{1+\chi}}{1+\chi} \right],$$

where E_0 is a date-0 expectations operator and $\beta \in (0,1)$ is a subjective discount factor. Notice the period utility function is defined over sequences of consumption $c_{j,t}$ relative to an *external* habit stock bc_{t-1}, where $c_{t-1} \equiv \int_0^1 c_{j,t-1} dj$ and is treated as given in the course of maximization. The parameter $b \in (0,1)$ measures the strength of external habit formation. Parameter $\sigma > 0$ governs the intertemporal elasticity of consumption and $\chi > 0$ the Frisch elasticity of labor supply. Preference shocks a_t affect all households symmetrically and follow the autoregressive process $\log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t}$, with $|\rho_a| < 1$ and $\varepsilon_{a,t} \sim$ i.i.d. $(0, \sigma_a^2)$.

Households enter each period with riskless one-period bond holdings $B_{j,t-1}$ that pay a gross nominal interest rate R_{t-1} at date t. They also provide labor services to firms at a competitive nominal wage rate W_t and, after production, receive dividends $\Phi_{j,t}$ from ownership of those firms. The period-t budget constraint is

$$P_t c_{j,t} + B_{j,t} \leq R_{t-1} B_{j,t-1} + (1-\tau) W_t h_{j,t} + \Phi_{j,t} + T_{j,t},$$

where $\tau \in (0,1)$ is a labor-income tax rate (calibrated to erase the steady-state distortions arising from market power and consumption externalities), and $T_{j,t}$ is a lump-sum government transfer. Sequences $\{c_{j,t}, h_{j,t}, B_{j,t}\}_{t=0}^{\infty}$ are chosen to maximize $V_{j,0}$ subject to the budget constraint and a no-Ponzi requirement, taking as given $\{a_t, c_{t-1}, P_t, R_{t-1}, W_t, \Phi_{j,t}, T_{j,t}\}_{t=0}^{\infty}$ and initial assets $B_{j,-1}$. The first-order conditions satisfy

$$1 = \beta E_t \frac{R_t}{\pi_{t+1}} \frac{a_{t+1}}{a_t} \left(\frac{c_{j,t} - bc_{t-1}}{c_{j,t+1} - bc_t} \right)^{\sigma},$$

$$h_{j,t}^{\chi} (c_{j,t} - bc_{t-1})^{\sigma} = w_t (1-\tau),$$

where $w_t \equiv W_t / P_t$ is the real wage and $\pi_t \equiv P_t / P_{t-1}$ is the gross inflation rate.

D.2. Firms

Good i is produced by a monopolistically competitive firm with technology $y_t(i) = z_t h_t(i)$, where $y_t(i)$ is the output of firm i and $h_t(i)$ its use of labor. Technology shocks z_t are common to all firms and follow $\log z_t = \rho_z \log z_{t-1} + \varepsilon_{z,t}$, with $|\rho_z| < 1$ and $\varepsilon_{z,t} \sim$ i.i.d. $(0, \sigma_z^2)$.

Firms maximize the present value of profit subject to

$$c_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta} c_t,$$

a market demand curve obtained by integrating $c_{j,t}(i)$ over all $j \in [0,1]$ households. Firms stand ready to meet demand at the posted price, so $z_t h_t(i) \geq c_t(i)$ for all $t \geq 0$. Individual prices may be reset every period, but at a cost. Specifically, firms pay adjustment costs of the form $(\alpha/2)\left[P_t(i)/\pi P_{t-1}(i) - 1\right]^2 y_t$, measured in units of aggregate output $y_t \equiv \int_0^1 y_t(i)di$, anytime growth in $P_t(i)$ deviates from the long-run mean inflation rate π. The constant $\alpha \geq 0$ determines the size of price adjustment costs.

The Lagrangian of firm i's maximization problem is

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} q_{0,t} \Bigg\{ & P_t(i)c_t(i) - W_t h_t(i) - \frac{\alpha}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 P_t y_t \\ & + P_t mc_t(i)\left[z_t h_t(i) - c_t(i)\right] + P_t \nu_t(i)\left[\left(\frac{P_t(i)}{P_t}\right)^{-\eta} c_t - c_t(i)\right]\Bigg\},\end{aligned}$$

where $q_{0,t}$ is a stochastic discount factor.[1] Sequences $\{h_t(i), c_t(i), P_t(i)\}_{t=0}^{\infty}$ are chosen to maximize $\mathcal{L}$, taking as given $\{q_{0,t}, W_t, P_t, y_t, z_t, c_t\}_{t=0}^{\infty}$ and the initial value $P_{-1}(i)$. The first-order conditions are

$$w_t = mc_t(i) z_t,$$

$$\nu_t(i) = \frac{P_t(i)}{P_t} - mc_t(i),$$

$$\begin{aligned} c_t(i) = \eta \nu_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\eta-1} c_t + \alpha\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)\frac{P_t y_t}{\pi P_{t-1}(i)} \\ - \alpha E_t \frac{q_{0,t+1}}{q_{0,t}} \pi_{t+1}\left(\frac{P_{t+1}(i)}{\pi P_t(i)} - 1\right)\frac{P_{t+1}(i) P_t y_{t+1}}{\pi P_t(i)^2}.\end{aligned}$$

D.3. Government

The government has a dual role in the model. First, it taxes labor income and remits the proceeds to households as lump-sum transfers, so $\tau W_t h_{j,t} = T_{j,t}$ for all $j \in [0,1]$. Second, it conducts monetary policy by adjusting R_t. Policy outcomes are optimal in that they maximize (under commitment or discretion) a second order approximation to $V_0 \equiv \int_0^1 V_{j,0} dj$.

[1] In equilibrium the stochastic discount factor satisfies $q_{0,t} P_t = \beta^t a_t (c_t - bc_{t-1})^{-\sigma}$.

D.4. Competitive Equilibrium

In a symmetric equilibrium households make identical spending and labor supply choices and firms charge the same price. It follows that subscript j and argument i can be dropped from the constraints and optimality conditions. Equilibrium also requires imposing relevant market-clearing conditions. Balancing the supply and demand for labor means $\int_0^1 h_{j,t}dj = \int_0^1 h_t(i)di \equiv h_t$ for $t \geq 0$. In product markets, supply of the final good equals demand from consumption plus resources spent on adjustment costs, so $y_t = c_t + (\alpha/2)(\pi_t/\pi - 1)^2 y_t$.

For completeness, the full set of symmetric equilibrium conditions are

$$
\begin{aligned}
h_t^{\chi} x_t^{\sigma} &= w_t(1-\tau), \\
a_t x_t^{-\sigma} &= \beta R_t E_t \left(a_{t+1} x_{t+1}^{-\sigma} / \pi_{t+1} \right), \\
x_t &= c_t - b c_{t-1}, \\
w_t &= mc_t z_t, \\
\nu_t &= 1 - mc_t, \\
c_t &= \eta \nu_t c_t + \alpha \left(\frac{\pi_t}{\pi} - 1 \right) \left(\frac{\pi_t}{\pi} \right) y_t - \alpha \beta E_t \left(\frac{a_{t+1} x_{t+1}^{-\sigma}}{a_t x_t^{-\sigma}} \right) \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \left(\frac{\pi_{t+1}}{\pi} \right) y_{t+1}, \\
y_t &= z_t h_t, \\
y_t &= c_t + (\alpha/2)(\pi_t/\pi - 1)^2 y_t,
\end{aligned}
$$

where I have defined $x_t \equiv c_t - b c_{t-1}$ the level of habit-adjusted aggregate consumption in equilibrium. Formally, a symmetric equilibrium is a set of processes $\{c_t, x_t, h_t, w_t, mc_t, \nu_t, y_t, \pi_t\}_{t=0}^{\infty}$ that satisfies the above equations, given a sequence of nominal interest rates $\{R_t\}_{t=0}^{\infty}$, initial conditions c_{-1}, and exogenous stochastic processes $\{a_t, z_t\}_{t=0}^{\infty}$.

D.5. Log-linear Approximation

To examine the welfare implications of the model with aggregate consumption habits, I first take log-linear approximations of the symmetric equilibrium conditions around the deterministic steady-state equilibrium. After substituting out consumption, work hours, and the real wage, the system of linear expectational difference equations becomes

$$\hat{x}_t = E_t\hat{x}_{t+1} - (1/\sigma)[\hat{R}_t - E_t\hat{\pi}_{t+1} - (1-\rho_a)\hat{a}_t], \tag{D.1}$$

$$\hat{x}_t = (1/(1-b))\hat{y}_t - (b/(1-b))\hat{y}_{t-1}, \tag{D.2}$$

$$\hat{mc}_t = \sigma\hat{x}_t + \chi\hat{y}_t - (1+\chi)\hat{z}_t, \tag{D.3}$$

$$\hat{\nu}_t = -(\eta-1)\hat{mc}_t, \tag{D.4}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} - (1/\alpha)\hat{\nu}_t. \tag{D.5}$$

To a first-order approximation, the aggregate demand component of this model, characterized by (D.1)–(D.3), is identical to the demand-side component of the deep habits model given by equations (M-1)–(M-3) in section 3 of the manuscript. In other words, deep and aggregate consumption habits have identical implications for aggregate demand *in equilibrium*. Where the two models differ is with regard to aggregate supply. Unlike the deep habits specification, aggregate consumption habits have no direct impact on inflation dynamics. Substituting (D.4) into (D.5) yields $\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + ((\eta-1)/\alpha)\,\hat{mc}_t$, which is the canonical New Keynesian Phillips Curve linking inflation to current and expected future marginal cost.