Appendix to "Inferring Monetary Policy Objectives with a Partially Observed State"

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Appendix A. The Partial Information Model

We provide a comprehensive derivation of the empirical state-space model introduced in section 4 of the manuscript. For the sake of completeness, we begin by restating the equations.

$$
\begin{aligned}
y_t &= \phi y_{t+1|t} + (1-\phi)[\beta y_{t-1} + (1-\beta) y_{t-2}] - \sigma(i_t - \pi_{t+1|t}) + \varepsilon_{y,t} && \text{(A.1)}\\
\pi_t &= \alpha \pi_{t+1|t} + (1-\alpha)\pi_{t-1} + \kappa(y_t - y_t^n) + \varepsilon_{\pi,t} && \text{(A.2)}\\
y_t^n &= \gamma y_{t-1}^n + \varepsilon_{n,t} + \eta_y \varepsilon_{y,t} && \text{(A.3)}\\
u_t - u_t^n &= -\chi(y_t - y_t^n) && \text{(A.4)}\\
\mathcal{L}_t &= E_t(1-\delta)\sum_{j=0}^{\infty}\delta^j \left[(\pi_{t+j} - \pi_{t+j}^*)^2 + \lambda_y (y_{t+j} - y_{t+j}^n)^2 + \lambda_i (i_{t+j} - i_{t+j-1})^2\right] && \text{(A.5)}\\
\pi_t^* &= \omega \pi_{t-1}^* + d(\pi_{t-1} - \pi_{t-1}^*) && \text{(A.6)}\\
\Delta y_t^o &= y_t - y_{t-1} + v_{g,t} && \text{(A.7)}\\
\pi_t^o &= \pi_t + v_{p,t} && \text{(A.8)}\\
v_{g,t} &= \rho_g v_{g,t-1} + \varepsilon_{g,t} && \text{(A.9)}\\
v_{p,t} &= \rho_p v_{p,t-1} + \varepsilon_{p,t} && \text{(A.10)}\\
\Delta u_t &= -\chi(y_t - y_t^n) + \chi(y_{t-1} - y_{t-1}^n) + \varepsilon_{u,t} && \text{(A.11)}
\end{aligned}
$$

In (A.1)–(A.11), y_t and y_t^n are real and natural output, u_t and u_t^n are the actual and natural rates of unemployment, i_t is the nominal interest rate, π_t is the inflation rate, π_t^* is the central bank's inflation target, $\varepsilon_{y,t}$ is a demand shock, $\varepsilon_{\pi,t}$ is a cost-push shock, $\varepsilon_{n,t}$ is a productivity shock, $\varepsilon_{u,t}$ is the innovation to natural unemployment, Δy_t^o and π_t^o are noisy measures of output growth and inflation, $v_{g,t}$ and $v_{p,t}$ are the noise components, and $\varepsilon_{g,t}$ and $\varepsilon_{p,t}$ are the innovations to those components. For any variable z_t, $z_{\tau|t}$ denotes $E[z_\tau|\Omega_t]$, the expected value of z_τ conditional on date-t information Ω_t. $\mathcal{L}_t$ is the policy loss function.

Define $X_t = [\varepsilon_{y,t}\ \varepsilon_{\pi,t}\ y_t^n\ \varepsilon_{u,t}\ v_{p,t}\ v_{g,t}\ y_{t-1}^n\ y_{t-1}\ y_{t-2}\ \pi_{t-1}\ \pi_{t-1}^*\ i_{t-1}]'$ the (12×1) vector of date-t predetermined variables, $x_t = [y_t\ \pi_t\ \pi_t^*\ \Delta u_t]'$ the (4×1) vector of date-t forward-looking variables, and $\varepsilon_{t+1} = [\varepsilon_{y,t+1}\ \varepsilon_{\pi,t+1}\ \varepsilon_{n,t+1}\ \varepsilon_{u,t+1}\ \varepsilon_{p,t+1}\ \varepsilon_{g,t+1}]'$ the (6×1) vector of guassian shocks with covariance matrix Σ.

Our first task is to write (A.1)–(A.11) in terms of X_t and x_t:

$$\begin{bmatrix} X_{t+1} \\ \Gamma x_{t+1|t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + Bi_t + \begin{bmatrix} N\varepsilon_{t+1} \\ \mathbf{0}_{4\times 1} \end{bmatrix}, \tag{A.12}$$

where A^1, A^2, B, Γ, N, and covariance matrix Σ are given by

$$A^1 = \begin{bmatrix} e_0 \\ e_0 \\ \gamma e_3 \\ e_0 \\ \rho_p e_5 \\ \rho_g e_6 \\ e_3 \\ e_{13} \\ e_8 \\ e_{14} \\ e_{15} \\ e_0 \\ e_{13} - (1-\phi)[\beta e_8 + (1-\beta)e_9] - e_1 \\ e_{14} - (1-\alpha)e_{10} - \kappa(e_{13} - e_3) - e_2 \\ -e_{15} + \omega e_{11} + d(e_{10} - e_{11}) \\ -e_{16} - \chi(e_{13} - e_3) + \chi(e_8 - e_7) + e_4 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \sigma \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} \phi & \sigma & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad A^2 = \left[\begin{array}{c|c} \mathbf{0}_{12\times 12} & \mathbf{0}_{12\times 4} \\ \hline \mathbf{0}_{4\times 12} & \mathbf{0}_{4\times 4} \end{array}\right]$$

$$N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \eta_y & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sigma_y^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_\pi^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_n^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_u^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_p^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_g^2 \end{bmatrix},$$

where e_j, $j = 0, 1, ..., 16$, denotes a 1×16 row vector with element j equal to one and all other elements equal to zero (for $j = 0$, $e_j = \mathbf{0}_{1\times 16}$).

The next task is to express the variables observed by the policymaker and private agents as functions of the right-hand-side variables in (A.12). Let $Z_t = [\Delta y_t^o \; \pi_t^o \; \Delta u_t]'$. Then

$$Z_t = [D_1 \; D_2] \begin{bmatrix} X_t \\ x_t \end{bmatrix}, \tag{A.13}$$

where D_1 and D_2 are defined as

$$D_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad D_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The third task is to determine the optimal setting of the interest rate under discretion. It is useful to first write the loss function in terms of X_t and x_t. Let $Y_t = [(\pi_t - \pi_t^*) \ \ (y_t - y_t^n) \ \ (i_t - i_{t-1})]'$ be the variables appearing in the loss function. Then

$$Y_t = C^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + C^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + C_i i_t,$$

where C^1, C^2, and C_i are

$$C^1 = \begin{bmatrix} e_{14} - e_{15} \\ e_{13} - e_3 \\ -e_{12} \end{bmatrix} \qquad C^2 = \left[\ \mathbf{0}_{3\times 12} \ \middle| \ \mathbf{0}_{3\times 4} \ \right] \qquad C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

The loss function may then be written as

$$\mathcal{L}_0 = E\left[(1-\delta) \sum_{t=0}^{\infty} \delta^t Y_t' W Y_t \middle| \Omega_0 \right], \tag{A.14}$$

where W is a diagonal matrix with non-zero elements $(1, \lambda_y, \lambda_i)$. Optimal discretion implies

$$i_t \ = \ F X_{t|t}, \tag{A.15}$$

where F is the matrix that solves the Ricatti equation characterizing optimal policy.

Our fourth task is to characterize expectations under optimal policy. Estimates of the forward-looking variables are related to estimates of the predetermined variables by

$$x_{t|t} \quad = \quad GX_{t|t}, \tag{A.16}$$

where G is a fixed point of the relation

$$G = (A_{22} - \Gamma G A_{12})^{-1}[(\Gamma G A_{11} - A_{21}) + (\Gamma G B_1 - B_2)F],$$

and $\{A_{11}, A_{12}, A_{21}, A_{22}, B_1, B_2\}$ are partitions of $A \equiv A^1 + A^2$ and B conformable to X_t and x_t. It follows that the relationship between forward-looking and predetermined variables is

$$x_t = G^1 X_t + G^2 X_{t|t}, \tag{A.17}$$

where G^1 and G^2 satisfy $G^1 = -(A^1_{22})^{-1} A^1_{21}$ and $G^2 = G - G^1$. It also follows that under optimal discretion, the evolution of the predetermined variables is governed by

$$X_{t+1} = HX_t + JX_{t|t} + N\varepsilon_{t+1}, \tag{A.18}$$

where matrices H and J satisfy $H = A^1_{11} + A^1_{12} G^1$ and $J = A^1_{12} G^2 + A^2_{11} + A^2_{12} G + B_1 F$.

Our fifth task is to explain how agents derive $X_{t|t}$. To accomplish this we need an indicator with the property that its innovation is a linear function of the forecast error $X_t - X_{t|t-1}$. Note that (A.13) and (A.17) imply that Z_t does not meet this standard due to the contemporaneous effect of $X_{t|t}$ on Z_t. Thus, we create an ideal indicator given by

$$\overline{Z}_t \equiv Z_t - MX_{t|t} = LX_t, \tag{A.19}$$

where $L = D_1 + D_2 G^1$ and $M = D_2 G^2$. We can then express the optimal prediction of X_t

in terms of the (steady-state) Kalman filter as follows:

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + K(\overline{Z}_t - \overline{Z}_{t|t-1}) \\ &= X_{t|t-1} + KL(X_t - X_{t|t-1}), \end{aligned} \tag{A.20}$$

where the (12×3) gain matrix K must be determined.

To find K, reformulate the problem in terms of prediction errors so that it admits a state-space form. Let $\tilde{X}_t \equiv X_t - X_{t|t-1}$ and $\tilde{Z}_t \equiv Z_t - Z_{t|t-1}$, and write (A.13) and (A.18) as

$$\begin{aligned} Z_t - Z_{t|t-1} &= L(X_t - X_{t|t-1}) + M(X_{t|t} - X_{t|t-1}) \\ \tilde{Z}_t &= (I + MK)L\tilde{X}_t = R\tilde{X}_t \end{aligned} \tag{A.21}$$

and

$$\begin{aligned} X_{t+1} - X_{t+1|t} &= HX_t + JX_{t|t} + N\varepsilon_{t+1} - HX_{t|t} - JX_{t|t} \\ &= H(X_t - X_{t|t}) + N\varepsilon_{t+1} \\ &= H(X_t - X_{t|t-1} - KL(X_t - X_{t|t-1})) + N\varepsilon_{t+1} \\ &= H(I - KL)(X_t - X_{t|t-1}) + N\varepsilon_{t+1} \\ \tilde{X}_{t+1} &= T\tilde{X}_t + N\varepsilon_{t+1}, \end{aligned} \tag{A.22}$$

where we have made use of (A.20) and defined $R \equiv (I + MK)L$ and $T \equiv H(I - KL)$.

Eqs. (A.22) and (A.21) are now the state and measurement equations for a standard Kalman-filter problem with $\tilde{X}_t$ as the unobserved variable and $\tilde{Z}_t$ as the observed variable. It follows that the prediction equation for $\tilde{X}_t$ is given by the standard formula for updating a linear projection

$$\tilde{X}_{t|t} = PR'(RPR')^{-1}R\tilde{X}_t, \tag{A.23}$$

where we use the fact that $\tilde{X}_{t|t-1} = 0$. $P \equiv \text{Cov}[\tilde{X}_t - \tilde{X}_{t|t-1}] = \text{Cov}[\tilde{X}_t] = \text{Cov}[X_t - X_{t|t-1}]$ is the (12×12) covariance matrix of the prediction errors for $\tilde{X}_t$, which are the same as the prediction errors for X_t since $\tilde{X}_{t|t-1} = 0$. Rewrite (A.23) in terms of X_t and $X_{t|t-1}$ to obtain

$$X_{t|t} = X_{t|t-1} + PR'(RPR')^{-1}R(X_t - X_{t|t-1}). \tag{A.24}$$

A comparison (A.20) and (A.24) shows that the Kalman gain matrix must equal

$$K = PL'(LPL')^{-1}, \tag{A.25}$$

where it remains to determine P. From (A.22) we get

$$\begin{aligned}
\text{Cov}[\tilde{X}_{t+1}] \equiv P = TPT' + N\Sigma N' &= H(I - KL)P(I - KL)'H' + N\Sigma N' \\
&= H(P - KLP)(I - L'K')H' + N\Sigma N' \\
&= H[P - PL'K' - KLP + KLPL'K']H' + N\Sigma N' \\
&= H[P - PL'K' - KLP + PL'K']H' + N\Sigma N' \\
&= H[P - KLP]H' + N\Sigma N' \\
P &= H[P - PL'(LPL')^{-1}LP]H' + N\Sigma N'. \qquad \text{(A.26)}
\end{aligned}$$

Therefore, P is defined as the fixed point of (A.26).

The final task is to derive the augmented state-space model that we take to the data. The *transition equation* is

$$\mathbf{s}_{t+1} = \mathbf{M}\mathbf{s}_t + \mathbf{N}\boldsymbol{\varepsilon}_{t+1}. \tag{A.27}$$

The (24×1) state vector is $\mathbf{s}_t \equiv [X_t' \; X_{t|t-1}']'$ and the (6×1) vector of shocks is $\boldsymbol{\varepsilon}_{t+1} =$

$[\varepsilon_{y,t+1}\ \varepsilon_{\pi,t+1}\ \varepsilon_{n,t+1}\ \varepsilon_{u,t+1}\ \varepsilon_{p,t+1}\ \varepsilon_{g,t+1}]'$. The (24×24) and (24×6) matrices $\mathbf{M}$ and $\mathbf{N}$ are

$$\mathbf{M} = \begin{bmatrix} H + JKL & J(I - KL) \\ (H+J)KL & (H+J)(I-KL) \end{bmatrix} \qquad \mathbf{N} = \begin{bmatrix} N \\ \mathbf{0}_{12\times 6} \end{bmatrix}.$$

The *measurement equation* is

$$\mathbf{y}_t = \mathbf{T}\mathbf{s}_t + \mathbf{u}_t, \tag{A.28}$$

where $\mathbf{y}_t \equiv [Z_t'\ i_t^o\ \Delta y_t\ \pi_t]'$ and $\mathbf{u}_t \equiv [0\ 0\ 0\ u_{i,t}\ 0\ 0]'$. Let $d_t \equiv [i_t^o\ \Delta y_t\ \pi_t]'$. Then

$$\begin{aligned} d_t &= S \begin{bmatrix} X_t \\ x_t \end{bmatrix} + S_i i_t + [u_{i,t}\ 0\ 0]' \\ &= S \begin{bmatrix} I & \mathbf{0}_{12\times 4} \\ G^1 & G^2 \end{bmatrix} \begin{bmatrix} X_t \\ X_{t|t} \end{bmatrix} + S_i F X_{t|t} + [u_{i,t}\ 0\ 0]' \\ &= \left(S \begin{bmatrix} I & \mathbf{0}_{12\times 4} \\ G^1 & G^2 \end{bmatrix} \begin{bmatrix} I & \mathbf{0}_{12\times 4} \\ KL & I - KL \end{bmatrix} + S_i F [KL \quad I - KL] \right) \begin{bmatrix} X_t \\ X_{t|t-1} \end{bmatrix} + \begin{bmatrix} u_{i,t} \\ 0 \\ 0 \end{bmatrix} \\ &= \Gamma_d \begin{bmatrix} X_t \\ X_{t|t-1} \end{bmatrix} + [u_{i,t}\ 0\ 0]', \end{aligned} \tag{A.29}$$

where matrices S and S_i are given by

$$S = \begin{bmatrix} e_0 \\ e_{13} - e_8 \\ e_{14} \end{bmatrix} \qquad S_i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Similarly, the indicators Z_t can be expressed as

$$\begin{aligned} Z_t &= LX_t + MX_{t|t} \\ &= LX_t + M(X_{t|t-1} + KL(X_t - X_{t|t-1})) \\ &= [(I+MK)L \quad M(I-KL)] \begin{bmatrix} X_t \\ X_{t|t-1} \end{bmatrix} = \Gamma_z \begin{bmatrix} X_t \\ X_{t|t-1} \end{bmatrix}. \end{aligned} \tag{A.30}$$

Stacking (A.30) and (A.29) yields (A.28) with (6×24) matrix $\mathbf{T}$ defined as

$$\mathbf{T} = \begin{bmatrix} \Gamma_z \\ \Gamma_d \end{bmatrix}.$$

Appendix B. The Complete Information Model

Under complete information agents observe all of the variables comprising X_t and x_t. The behavioral relationships and the loss function given by (M-1)–(M-6) are exactly the same as in the partial information model. To characterize the dynamics of the complete information model, we appeal to the certainty equivalence principle. Specifically, optimal policy under partial information is identical to the one under full information, except that one responds to an efficient estimate of the state rather than the actual state. It follows that the recursive equilibrium can be found by replacing $X_{t|t}$ with X_t in (A.15), (A.17), and (A.18)

$$i_t = FX_t \tag{B.1}$$

$$x_t = GX_t \tag{B.2}$$

$$X_{t+1} = (H+J)X_t + N\varepsilon_{t+1}, \tag{B.3}$$

where $G = G^1 + G^2$ and $H + J = A_{11} + A_{12}G + B_1F$.

It is straightforward to express the equilibrium in state-space form so that the parameters can be estimated using maximum likelihood. Since beliefs about economic conditions are always correct, there is no need to augment the state with efficient forecasts of the predetermined variables as in (A.27). It follows that the *transition equation* is given by (B.3).

The *measurement equation* links the econometrician's observed variables to X_t. As explained in section 4.1 of the manuscript, the variables relevant for estimation are $\tilde{d}_t \equiv [\Delta u_t \; i_t^o \; \Delta y_t \; \pi_t]'$. Under full information the model makes no distinction between the true values of output growth and inflation and the observable concepts seen in real time, so the shocks $v_{g,t}$ and $v_{p,t}$ in (A.7) and (A.8) equal zero every period. As a result, we discard the real-time data on these variables and estimate the model using only the final published data. Specifically, the measurement equation takes the form

$$\begin{aligned} \tilde{d}_t &= \tilde{S} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \tilde{S}_i i_t + [0 \; u_{i,t} \; 0 \; 0]' \\ &= \left(\tilde{S} \begin{bmatrix} I \\ G \end{bmatrix} + \tilde{S}_i F \right) X_t + [0 \; u_{i,t} \; 0 \; 0]', \end{aligned} \tag{B.4}$$

where

$$\tilde{S} = \begin{bmatrix} e_{16} \\ e_0 \\ e_{13} - e_8 \\ e_{14} \end{bmatrix} \qquad \tilde{S}_i = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

The system given by (B.3) and (B.4) can then be used to evaluate the log-likelihood function.

Appendix C. An Estimate of the Gain Matrix

In this appendix we clarify how the indicator variables are used by private agents and the central bank in revising forecasts of the state. The information is provided by estimates of an updating matrix U, which is a function of the Kalman gain matrix K described in section 3.2 of the manuscript. Recall that inferences about X_t are updated according to $X_{t|t} = X_{t|t-1} + KL(X_t - X_{t|t-1})$. Rewriting this forecasting equation so that it depends on innovations to the indicators Z_t instead of innovations to X_t yields $X_{t|t} = X_{t|t-1} + U(Z_t - Z_{t|t-1})$, where it can be shown that $U \equiv K(I + MK)^{-1}$. The $(i{,}j)$ element of U is the weight placed on innovations to the j^{th} indicator on forecasts of the i^{th} state variable. Given the partial information estimates reported in Table 3 of the manuscript, the updating matrices for the first and second subsamples are

$$U_1 = \left(\begin{array}{c|ccc}
 & \Delta y_t^o & \pi_t^o & \Delta u_t \\
\hline
\varepsilon_{y,t|t} & \underset{(0.1574)}{0.2356} & \underset{(0.0393)}{0.0000} & \underset{(0.2255)}{-0.0648} \\
\varepsilon_{\pi,t|t} & \underset{(0.0044)}{-0.0002} & \underset{(0.0357)}{0.3400} & \underset{(0.0067)}{0.0001} \\
y^n_{t|t} & \underset{(0.0737)}{0.1857} & \underset{(0.0008)}{-0.0000} & \underset{(0.3382)}{0.4227} \\
\varepsilon_{u,t|t} & \underset{(0.1426)}{0.1438} & \underset{(0.0323)}{-0.0000} & \underset{(0.3222)}{0.7987} \\
v_{p,t|t} & \underset{(0.0043)}{-0.0002} & \underset{(0.0508)}{0.3336} & \underset{(0.0066)}{0.0001} \\
v_{g,t|t} & \underset{(0.1561)}{0.4139} & \underset{(0.0376)}{0.0000} & \underset{(0.2688)}{0.3554} \\
y^n_{t-1|t} & \underset{(0.1594)}{-0.0826} & \underset{(0.0347)}{-0.0000} & \underset{(0.4796)}{0.3334} \\
y_{t-1|t} & \underset{(0.1650)}{-0.0310} & \underset{(0.0401)}{-0.0000} & \underset{(0.3146)}{0.5283} \\
y_{t-2|t} & \underset{(0.2011)}{0.0066} & \underset{(0.0493)}{-0.0001} & \underset{(0.3289)}{0.7477} \\
\pi_{t-1|t} & \underset{(0.0008)}{-0.0000} & \underset{(0.0213)}{0.0640} & \underset{(0.0013)}{0.0000} \\
\pi^*_{t-1|t} & \underset{(0.0000)}{-0.0000} & \underset{(0.0001)}{0.0003} & \underset{(0.0000)}{0.0000} \\
i_{t-1|t} & \underset{(0.0000)}{0.0000} & \underset{(0.0000)}{0.0000} & \underset{(0.0000)}{0.0000}
\end{array}\right)$$

$$U_2 = \left(\begin{array}{c|ccc}
 & \Delta y_t^o & \pi_t^o & \Delta u_t \\
\hline
\varepsilon_{y,t|t} & \underset{(0.0143)}{0.2166} & \underset{(0.0036)}{0.0102} & \underset{(0.0327)}{-0.2070} \\
\varepsilon_{\pi,t|t} & \underset{(0.0037)}{-0.0052} & \underset{(0.0893)}{0.3945} & \underset{(0.0156)}{0.0263} \\
y^n_{t|t} & \underset{(0.0532)}{0.3519} & \underset{(0.0080)}{-0.0205} & \underset{(0.2927)}{1.5941} \\
\varepsilon_{u,t|t} & \underset{(0.0147)}{0.0236} & \underset{(0.0004)}{-0.0004} & \underset{(0.0693)}{0.1128} \\
v_{p,t|t} & \underset{(0.0047)}{-0.0048} & \underset{(0.0593)}{0.3509} & \underset{(0.0172)}{0.0224} \\
v_{g,t|t} & \underset{(0.0335)}{0.3925} & \underset{(0.0026)}{0.0064} & \underset{(0.0801)}{0.5815} \\
y^n_{t-1|t} & \underset{(0.0194)}{-0.1868} & \underset{(0.0083)}{-0.0153} & \underset{(0.1090)}{-0.4048} \\
y_{t-1|t} & \underset{(0.0186)}{-0.0805} & \underset{(0.0021)}{-0.0044} & \underset{(0.0655)}{0.1935} \\
y_{t-2|t} & \underset{(0.0161)}{-0.0818} & \underset{(0.0023)}{-0.0054} & \underset{(0.0794)}{0.2537} \\
\pi_{t-1|t} & \underset{(0.0011)}{-0.0006} & \underset{(0.0510)}{0.1607} & \underset{(0.0100)}{0.0193} \\
\pi^*_{t-1|t} & \underset{(0.0001)}{0.0001} & \underset{(0.0007)}{0.0020} & \underset{(0.0007)}{0.0008} \\
i_{t-1|t} & \underset{(0.0000)}{0.0000} & \underset{(0.0000)}{0.0000} & \underset{(0.0000)}{0.0000}
\end{array}\right),$$

where standard errors (in parentheses) are found using the delta method.

Our estimates of U indicate that observed changes in the unemployment rate had very different effects on perceptions of demand shocks, cost-push shocks, and natural output. A unit innovation to Δu_t evidently caused agents to revise down their forecast of $\varepsilon_{y,t}$ by 0.06 percentage points before 1979 but 0.21 percentage points after 1979. The impact on forecasts of natural output was even more significant. Over the sample period agents updated their estimate of y_t^n by 0.42 percentage points in the pre-Volcker era and 1.59 percentage points thereafter. Observations on the unemployment rate, however, appear to have had little information content for cost-push shocks. Estimates of $\varepsilon_{\pi,t}$ were revised up by only 0.03 percentage points after 1979 and essentially unchanged before that.

The reason why agents relied heavily on Δu_t in forming estimates of demand shocks and natural output but very little in estimating cost-push shocks is because the signal-to-noise ratio implied by the semi-structural model is considerably higher for the former. This is a result of the strong contemporaneous linkage between unemployment and the output gap established by Okun's Law. In the partial information model the estimate of the Okun coefficient χ is around 0.40. The relationship with cost-push shocks, however, is much weaker since inflation only affects unemployment indirectly by shifting output via the real interest rate. Estimates of the slope coefficient σ in the IS equation are quite small both before and after 1979. It follows that changes in the real interest rate stemming from cost-push shocks will have limited effects on output and hence the unemployment rate.

Appendix D. Estimates of Target Inflation

Section 5.1 of the manuscript presents empirical evidence of a shift in the weights characterizing the Federal Reserve's loss function at the time of Volcker's appointment. In this appendix we turn our attention to a different aspect of the loss function, namely, the time-varying inflation target π_t^*. Our goal is to extract historical estimates of π_t^* from the observable macroeconomic data used to estimate the partial and complete information models. To that end, we apply the Kalman smoother described in section 5.3 of the manuscript to both models evaluated at their respective maximum-likelihood point estimates. In each case we regard the estimated model as the true data generating process and use the smoother to generate two-sided estimates of the unobserved inflation target from 1965:Q4 to 2010:Q1. The two filtered series along with the actual inflation rate are graphed together in Fig. I.

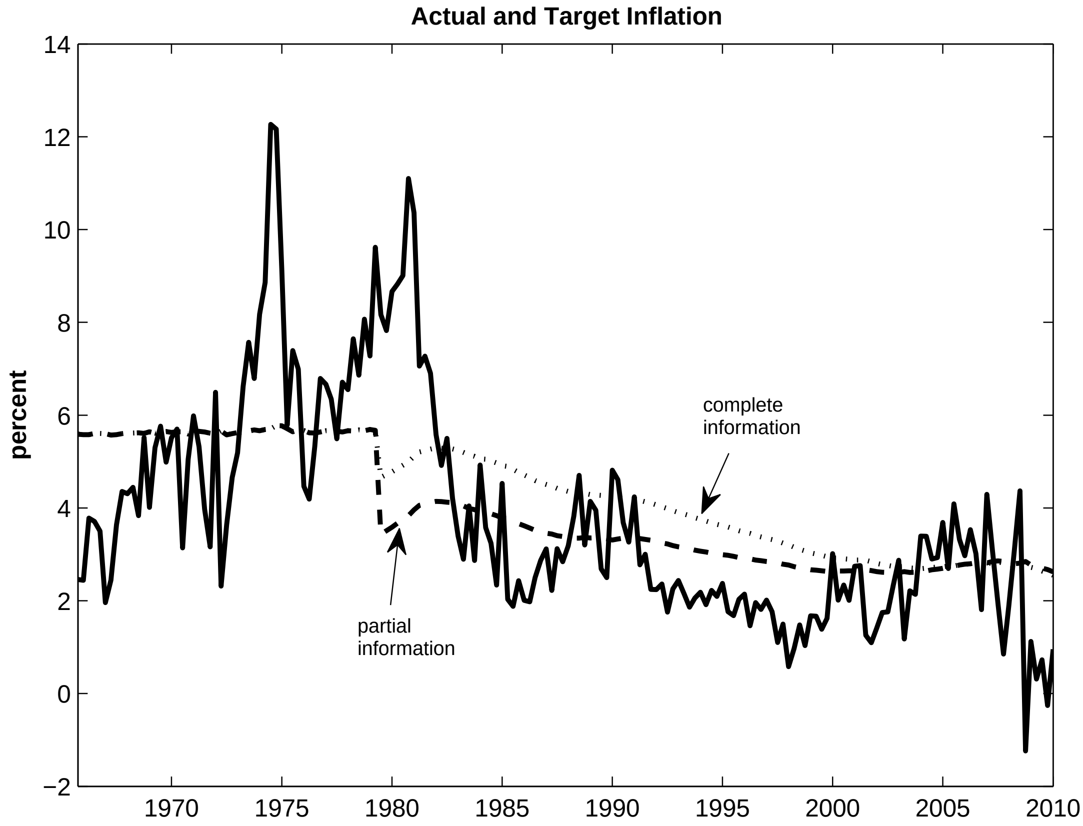

Fig. I. The historical inflation series (π_t, solid line) is plotted against the Federal Reserve's inflation target, π_t^*, as implied by the estimated partial information model (dashed line) and the estimated complete information model (dotted line).

The partial and complete information models indicate that prior to 1979, the Fed's implicit inflation target was basically flat at around 5.6 percent even though actual inflation trended sharply higher during the same period. We trace this result to the pre-1979 estimates of ω, which converge to zero in both models. With the value of d fixed at 0.02, the absence of any serial persistence in the stochastic process for π_t^* means that the inflation target will vary little over the sample period.

The partial and complete information estimates of π_t^* split after the beginning of Volcker's term. According to the partial information model, there was a discrete drop in the inflation target of more than two percentage points in late 1979. The next two decades witnessed a gradual decline in π_t^* to about 2.5 percent that mirrored the downward trend in actual inflation. The post-1979 profile is quite different under complete information. According to this model, the initial drop in target inflation was only about one percentage point and, in turn, paved the way for a more rapid decline throughout the 1980s and 1990s.

Appendix E. A Constant Inflation Target

Some have argued that incorporating time variation in the central bank's inflation target can expose one's model to identification problems. A prominent example from the recent literature is the idea that the model might attribute persistence in the inflation data to drift in the target rather than lags in the behavioral equations. Another raises the possibility that the observed co-movement between output and inflation could potentially be explained by the type of policy tradeoffs reflected in the loss function weights or simply by procyclical variation in the inflation target. The point here is that a moveable inflation target will undoubtedly compete with other features of the model in an effort to maximize goodness-of-fit. Unfortunately, expanding one's model in ways that permit multiple interpretations of the data can make estimation of all the structural parameters a more difficult task.

To see whether the time-varying nature of π_t^* makes identification problematic, we re-estimate both the partial and complete information models under the assumption of a fixed inflation target. In practice this done by restricting $\omega = d = 0$ in (A.6). Since the observable data are de-meaned prior to estimation, fixing $\pi_t^* = 0$ also ensures that target inflation will correspond to the sample mean. The full set of estimation results for the periods before and after 1979 are reported in Table I.

For the pre-1979 period we see that the estimates under a constant inflation target are nearly identical to the benchmark estimates found in Table 3 of the manuscript. This result is not surprising given that ω, the persistence in the inflation target, converges to zero in the course of estimating the benchmark partial and complete information models. With the

Table I
Fixed inflation target

		Partial Information			Complete Information		
Parameter	Description	1965:Q4–1979:Q2	1979:Q3–2010:Q1	W	1965:Q4–1979:Q2	1979:Q3–2010:Q1	W
σ_y	*demand shock*	0.6457 (0.0507)	0.4631 (0.0245)	0.0012	0.6014 (0.0747)	0.2729 (0.0180)	0.0000
σ_π	*cost-push shock*	0.9691 (0.0799)	0.8741 (0.1292)	0.5318	0.8848 (0.1026)	0.5636 (0.0381)	0.0033
σ_n	*natural output shock*	0.1594 (0.2285)	0.5498 (0.0607)	0.0987	0.1901 (0.1071)	0.2885 (0.0457)	0.3980
σ_u	*natural unemployment shock*	0.2083 (0.0420)	0.0753 (0.0224)	0.0052	0.1543 (0.0821)	0.1677 (0.0200)	0.8743
σ_g	*output growth noise*	0.7681 (0.0734)	0.4754 (0.0313)	0.0002	—	—	—
σ_p	*inflation noise*	0.9685 (0.1009)	0.8808 (0.0637)	0.4623	—	—	—
σ_i	*interest rate shock*	0.4890 (0.0504)	0.8711 (0.0580)	0.0000	0.8211 (0.0814)	0.6732 (0.0578)	0.1385
ρ_g	*serial correlation in* σ_g	0.0092 (0.1438)	0.0105 (0.0915)	0.9941	—	—	—
ρ_p	*serial correlation in* σ_p	0.0973 (0.1404)	0.1201 (0.0934)	0.8928	—	—	—
ϕ	*expected future output*	0.3871 (0.0072)	0.3595 (0.0037)	0.0006	0.4425 (0.0611)	0.3726 (0.0053)	0.2543
β	*lagged output*	1.4194 (0.0071)	1.4867 (0.0055)	0.0000	0.9537 (0.0818)	1.4522 (0.0091)	0.0000
σ	*interest rate elasticity*	1.63e-6 (0.0048)	0.0012 (0.0004)	0.8049	0.1222 (0.0642)	0.0005 (0.0003)	0.0580
α	*expected future inflation*	0.5533 (0.0191)	0.2298 (0.1685)	0.0566	0.5652 (0.0295)	0.5220 (0.0157)	0.1958
κ	*output gap elasticity*	4.15e-5 (1.70e-5)	0.0236 (0.0139)	0.0889	0.0212 (0.0075)	0.0055 (0.0034)	0.0574
γ	*lagged natural output*	0.8082 (0.0535)	0.9051 (0.0189)	0.0880	0.6222 (0.0961)	0.9291 (0.0202)	0.0017
η_y	*demand shock feedback*	0.9095 (0.0527)	0.4287 (0.0883)	0.0000	1.1568 (0.1619)	1.2187 (0.1687)	0.7913
χ	*Okun coefficient*	0.4526 (0.0494)	0.3576 (0.0223)	0.0793	0.7078 (0.1397)	0.4585 (0.0389)	0.0856
λ_y	*output gap weight*	0.0339 (0.0627)	0.3534 (0.1201)	0.0183	$0^{\dagger}$	0.4327 (0.3190)	0.1750
λ_i	*interest-rate smoothing weight*	$0.001^{\dagger}$	1.8136 (0.5792)	0.0018	2.2225 (1.0594)	0.7387 (0.4617)	0.1992
ω	*inflation target persistence*	0.00^*	0.00^*	—	0.00^*	0.00^*	—
d	*inflation target accommodation*	0.00^*	0.00^*	—	0.00^*	0.00^*	—
δ	*loss discount factor*	0.996^*	0.996^*	—	0.996^*	0.996^*	—
$\ln \mathcal{L}$	*log likelihood*	−359.0360	−725.1390		−236.5486	−447.3827	

Notes: The table reports maximum-likelihood estimates of (M-1)–(M-6) and (I-1)–(I-3) under partial and complete information. Numbers in parentheses are standard errors. The columns labeled W contain the p-values of the Wald statistic for testing the null hypothesis of parameter stability. * denotes a value that is imposed prior to estimation. † denotes a value that lies on the boundary of the allowable parameter space.

value of d having already been fixed at 0.02, $\omega = 0$ implies that there will be almost no variation in π_t^* over the sample (see Fig. I in Appendix D). Thus at least for the pre-Volcker era, inserting a time-varying inflation target into the loss function does not appear to create any new identification problems.

Data from the post-1979 period evidently favors much greater persistence in the inflation target since the unrestricted estimates of ω are close to one in both the partial and complete information cases. That being said, imposing $\omega = d = 0$ does not alter the central results of our estimation. Parameters that are perhaps most affected by fixing the inflation target are the loss function weights, λ_i and λ_y. Under partial information holding π_t^* constant increases estimates of both weights relative to their benchmark values. In other words, the Federal Reserve's preference for inflation stability relative to its other two goals is somewhat diminished when the model prohibits variation in the inflation target. The magnitude of these changes, however, appear rather limited, so we conclude that the model's basic interpretation of the data is the same with or without a time-varying inflation target. Finally, notice that estimates of the parameter standard errors are also similar to the benchmark results. This suggests that relaxing the assumption of a fixed inflation target probably does not weaken identification of our model.

Appendix F. Robustness Checks

In this appendix we examine the robustness of our partial information estimates to changes in the processes governing inflation measurement shocks as well as the natural unemployment rate. The benchmark model described in section 2 of the manuscript treats the noise component of π_t^o as a stationary first-order autoregressive shock. In what follows we re-estimate the model conditional on a stationary ARMA(1,1) process for this component given by $v_{p,t} = \rho_p v_{p,t-1} + \varepsilon_{p,t} - \mu \varepsilon_{p,t-1}$ with $\varepsilon_{p,t} \sim i.i.d.\ N(0, \sigma_p^2)$. Our new specification nests the AR(1) arrangement as a special case ($\mu = 0$) but allows for the possibility that $v_{p,t}$ also has

Table II
Sensitivity analysis

		1979:Q3–2010:Q1	
Parameter	Description	$v_{p,t} \sim$ ARMA(1,1)	$u_t^n \sim$ AR(1)
σ_y	*demand shock*	0.4616 (0.0245)	0.4592 (0.0245)
σ_π	*cost-push shock*	0.8846 (0.1328)	0.9354 (0.1517)
σ_n	*natural output shock*	0.5749 (0.0702)	0.5519 (0.0720)
σ_u	*natural unemployment shock*	0.0784 (0.0240)	0.0749 (0.0258)
σ_g	*output growth noise*	0.4707 (0.0311)	0.4705 (0.0310)
σ_p	*inflation noise*	0.8673 (0.0619)	0.8650 (0.0599)
σ_i	*interest rate shock*	0.8413 (0.0566)	0.8399 (0.0556)
ρ_g	*serial correlation in* σ_g	−0.0151 (0.0927)	−0.0184 (0.0928)
ρ_p	*serial correlation in* σ_p	−0.4410 (0.3223)	0.1071 (0.0901)
μ	*moving average in* σ_p	0.6170 (0.2798)	—
ρ_n	*serial correlation in* σ_u	—	0.9915 (0.0094)
ϕ	*expected future output*	0.3577 (0.0040)	0.3574 (0.0041)
β	*lagged output*	1.4872 (0.0057)	1.4854 (0.0062)
σ	*interest rate elasticity*	0.0009 (0.0003)	0.0008 (0.0003)
α	*expected future inflation*	0.2147 (0.1696)	0.1490 (0.1868)
κ	*output gap elasticity*	0.0214 (0.0130)	0.0180 (0.0133)
γ	*lagged natural output*	0.9020 (0.0184)	0.9002 (0.0179)
η_y	*demand shock feedback*	0.4208 (0.0921)	0.4410 (0.0885)
χ	*Okun coefficient*	0.3436 (0.0242)	0.3569 (0.0280)
λ_y	*output gap weight*	0.2256 (0.0724)	0.1812 (0.0888)
λ_i	*interest-rate smoothing weight*	1.1437 (0.5376)	0.8362 (0.6219)
ω	*inflation target persistence*	0.9707 (0.0141)	0.9807 (0.0129)
d	*inflation target accommodation*	0.02*	0.02*
δ	*loss discount factor*	0.996*	0.996*
$\ln \mathcal{L}$	*log likelihood*	−720.8378	−722.0660

Notes: The table reports maximum-likelihood estimates of (M-1)–(M-6) and (I-1)–(I-3) under partial information. The first column of estimates allow inflation measurement shocks to follow the stationary ARMA(1,1) process $v_{p,t} = \rho_p v_{p,t-1} + \varepsilon_{p,t} - \mu\varepsilon_{p,t-1}$. The second column allows natural unemployment to follow the stationary AR(1) process $u_t^n = \rho_n u_{t-1}^n + \varepsilon_{u,t}$. Numbers in parentheses are standard errors. * denotes a value that is imposed prior to estimation.

a moving average element. The findings are reported in the first column of Table II.

Modifying the law of motion for $v_{p,t}$ affects estimates of ρ_p and μ but does not have a major impact on the remaining parameters nor does it significantly improve model fit. The point estimate of ρ_p is now -0.44 rather than 0.11, indicating that measurement shocks may actually exhibit negative serial correlation. Moreover, the estimate of the moving average coefficient μ is 0.62. Because the ARMA(1,1) specification nests the benchmark AR(1) arrangement, we can conduct a formal likelihood ratio test of the null hypothesis that $\mu = 0$. The p-value of the relevant chi-square statistic is 0.09, so we reject the null restriction at the 10 percent level but not at the 5 percent level.

In our original setup fluctuations in the natural rate of unemployment follow a pure random walk. Although we cite a number of empirical studies to justify this modeling choice in the manuscript, one potential drawback is that the observed unemployment rate inherits a unit root directly from u_t^n via the Okun's Law relationship (M-4). We now examine the implications of imposing stationarity on u_t by conditioning estimation on the assumption that natural unemployment follows an AR(1) process $u_t^n = \rho_n u_{t-1}^n + \varepsilon_{u,t}$ with $|\rho_n| < 1$ and $\varepsilon_{u,t} \sim i.i.d.\ N(0, \sigma_u^2)$. The estimates are reported in the second column of Table II.

Comparing these results to the ones in Table 3 of the manuscript reveals that our random walk assumption has little effect on the majority of parameter estimates and does not appear to harm model fit. The similarity between the two sets of results is perhaps not surprising given that the point estimate of ρ_n exceeds 0.99 and is not significantly different from one. Indeed, a likelihood ratio test of the restriction $\rho_n = 1$ cannot be rejected at standard significance levels. Thus while the assumption of a nonstationary unemployment rate may be theoretically undesirable, in practice we find that it does not compromise the central findings of our study.