

Appendix to “Generalized method of moments and inverse control”

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Appendix A. Representative agent model

In this section we describe the general equilibrium model estimated in section 3.3 of Givens and Salemi (2007). A variant of the prototype New Keynesian model expounded by Goodfriend and King (1997) and Rotemberg and Woodford (1997)), the model integrates staggered price-setting and monopolistic competition into an optimizing-agent framework. In what follows we discuss the assumptions concerning preferences and price setting and derive the key equilibrium conditions that give rise to the equations in the text.

A.1. The household sector

The economy is inhabited by a large number of identical households that make intertemporal consumption and saving decisions and supply labor to the production sector. The preferences of the representative household are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \{U(C_t - bC_{t-1}; u_t) - \nu(H_t)\} \quad (\text{A.1})$$

where U is a monotonic and strictly concave period utility function defined over sequences of

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consumption C_t relative to an internal habit stock bC_{t-1} . C_t is the following CES aggregator of differentiated goods:

$$C_t = \left(\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$$

and $P_t = \left(\int_0^1 p_t(i)^{1-\eta} di \right)^{\frac{1}{1-\eta}}$ is the aggregate price index, where $p_t(i)$ denotes the price of good $i \in [0, 1]$. The parameter $\eta > 1$ is the elasticity of substitution between alternative varieties while $b \in (0, 1)$ measures the degree of habit persistence. The stochastic variable u_t is a white-noise taste shock that generates exogenous variation in the marginal utility of income for given levels of consumption. The function ν denotes the period disutility of supplying work hours H_t and is strictly increasing and convex.

The household's flow budget constraint takes the following form:

$$C_t + \frac{B_t}{P_t} \leq W_t H_t + \frac{R_{t-1} B_{t-1}}{P_t} + \int_0^1 Div_t(i) di \quad (\text{A.2})$$

where B_{t-1} denotes the quantity of riskless, one-period bonds carried into period t and R_{t-1} is the corresponding gross nominal interest rate. $W_t H_t$ represents labor income and $Div_t(i)$ is a stream of real profits from ownership of firm i .

The representative household chooses an optimal plan $\{C_t, H_t, B_t\}_{t=0}^{\infty}$ by maximizing (A.1) subject to (A.2) taking as given the processes $\{P_t, R_t, W_t, Div_t(i) : i \in [0, 1]\}_{t=0}^{\infty}$ and the initial values B_{-1} , R_{-1} , and C_{-1} . The first-order conditions are given by

$$U_c(C_t - bC_{t-1}; u_t) - \beta b E_t U_c(C_{t+1} - bC_t; u_{t+1}) = \Lambda_t \quad (\text{A.3})$$

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} R_t \frac{P_t}{P_{t+1}} \right] \quad (\text{A.4})$$

$$W_t = \frac{\nu_H(H_t)}{\Lambda_t} \quad (\text{A.5})$$

where Λ_t is the Lagrange multiplier associated with (A.2). To obtain equation (21) in the text, we combine the log-linear approximations of (A.3) and (A.4) together with the equilibrium requirement $Y_t = C_t$.

A.2. The production sector

Monopolistically competitive firms produce differentiated goods using the technology

$$Y_t(i) = \exp(v_t) K_t(i)^\alpha H_t(i)^{1-\alpha} \quad (\text{A.6})$$

where $\alpha \in (0, 1)$ is the capital elasticity of output and v_t is a serially uncorrelated productivity disturbance. Although we assume that the *aggregate* capital stock is fixed at \bar{K} , capital and labor are perfectly mobile, enabling firms to adjust input quantities in a way that equalizes capital-to-labor ratios. Consequently, equilibrium will feature common real marginal costs per unit of output across industries, which can be expressed as

$$MC_t = \frac{W_t}{(1 - \alpha) \exp(v_t)} \left(\frac{H_t}{\bar{K}} \right)^\alpha. \quad (\text{A.7})$$

Sticky prices are modeled in the fashion of Calvo (1983). Firms face a constant probability $(1 - \varepsilon)$ in each period of realizing an opportunity to reset their price $p_t(i)$, independent of the time elapsed since their previous adjustment. Firms that do not reset optimally use the following indexation rule to update existing prices:

$$p_t(i) = \Pi_{t-1}^\gamma \times p_{t-1}(i) \quad (\text{A.8})$$

where $\Pi_t = P_t/P_{t-1}$ and $\gamma \in [0, 1]$ measures the degree of indexation to past inflation. Let \tilde{p}_t denote the optimal value of $p_t(i)$ chosen by all firms that adjust in period t . Firms select

\tilde{p}_t to maximize the present value of expected future real profits given by

$$E_t \sum_{j=0}^{\infty} (\varepsilon\beta)^j \frac{\Lambda_{t+j}}{\Lambda_t} Y_{t+j}(i) \left\{ \frac{\tilde{p}_t}{P_{t+j}} \left(\prod_{k=0}^{j-1} \Pi_{t+k}^\gamma \right) - MC_{t+j} \right\}. \quad (\text{A.9})$$

The first-order condition with respect to \tilde{p}_t can be expressed as

$$E_t \sum_{j=0}^{\infty} (\varepsilon\beta)^j \Lambda_{t+j} Y_{t+j}(i) \left\{ \frac{\tilde{p}_t}{P_t} \left(\prod_{k=1}^j \Pi_{t+k}^{-1} \right) \left(\prod_{k=0}^{j-1} \Pi_{t+k}^\gamma \right) - \frac{\eta}{\eta-1} MC_{t+j} \right\} = 0. \quad (\text{A.10})$$

Using the definition of the aggregate price index, it is clear that the evolution of the price level over time must satisfy

$$P_t^{1-\eta} = (1-\varepsilon)\tilde{p}_t^{1-\eta} + \varepsilon (\Pi_{t-1}^\gamma \times P_{t-1})^{1-\eta}. \quad (\text{A.11})$$

To obtain equation (22) in the text, we combine the log-linear approximations of (A.7), (A.10), and (A.11) together with the equilibrium requirement $Y_t = C_t$.

A.3. The flexible price equilibrium

To evaluate the welfare cost of alternative policies using a quadratic approximation to (A.1), it is necessary to track the dynamic behavior of the model's flexible price equilibrium. Suppose that all firms reset prices optimally every period ($\varepsilon \rightarrow 0$), implying that $p_t(i) = \tilde{p}_t = P_t$ for all $i \in [0, 1]$. It follows that equation (A.10) will collapse to the familiar markup condition, $MC_t = \frac{\eta-1}{\eta}$, and every firm will produce identical quantities.

Denote Y_t^n the flexible price level of output. It can be shown that Y_t^n is determined implicitly by the following condition:

$$\left(\frac{\eta-1}{\eta} \right) (1-\alpha) \exp(v_t) \left(\frac{\bar{K}}{H_t} \right)^\alpha = \frac{\nu_H(H_t)}{\Lambda_t} \quad (\text{A.12})$$

after eliminating H_t using the aggregate relationship $Y_t = \exp(v_t)\bar{K}^\alpha H_t^{1-\alpha}$. Equation (23) in the text is simply the log-linear approximation of (A.12).

Appendix B. A comparison of GMM and FIML

In this section we compare the performance of the GMM procedure developed by Givens and Salemi (2007) to that of a full information maximum likelihood (FIML) strategy that nests the control problem of the central bank within the estimation exercise of the econometrician.¹ The nested approach searches over values of the structural parameters and loss function weights for those that maximize the likelihood function while constraining the policy-rule coefficients to be those that minimize expected loss. Nesting the control problem requires one to compute optimal policy-rule coefficients for every set of structural parameters and loss function weights considered in the course of estimation. GMM, on the other hand, combines least squares normal equations with the first order conditions for optimal policy and, in a sense, moves gradually toward estimates that satisfy both criteria rather than fully satisfying the optimal policy criterion for each set of parameters considered.

In the paper, we demonstrated GMM on three different New Keynesian-style models. For a comparison of GMM with FIML, however, we focus exclusively on the forward-looking model studied in section 3.2. We feel that the marginal benefits of providing comparisons for the backward-looking and representative agent models are relatively small. The differences and similarities between GMM and FIML identified in the context of the forward-looking model are also likely to appear in the other two models.

B.1. Computation speed

We first compare GMM and FIML according to the computation time required by each procedure. Recall that GMM estimation of the forward-looking model requires a search over

¹Salemi (2006) and Dennis (2004) use the nested strategy to describe Federal Reserve policy between 1965 and 2001.

13 parameters: 7 parameters of the structural equations, 2 loss function weights, and 4 policy-rule coefficients. The reduced-form error covariance matrix, which is needed for computing the partial derivatives of expected loss, can be obtained at each step from the residual-error covariance matrix. Estimation by FIML, however, requires a search over 17 parameters: 7 parameters of the structural equations, 2 loss function weights, and 6 parameters that pin down the reduced-form error covariance matrix. The 4 policy-rule coefficients are determined internally as the solution to the loss minimization problem. Thus, estimation by GMM actually requires a search over fewer parameters than FIML. However, the GMM procedure must be repeated twice in order to obtain consistent estimates. In the first stage the identity matrix is used as the GMM weighting matrix. In the second stage the optimal weighting matrix is computed and then parameter estimation is repeated.

Given the asymmetries in the estimated parameter space, how much computation time is required by each procedure? When the optimality hypothesis is true, GMM and FIML require approximately the same amount of computation time. The average number of seconds per sample, computed across 20 trials each of sample sizes 100, 250, 500, and 1,000, was 303 seconds under GMM and 234 seconds under FIML.² When the optimality hypothesis is false, GMM requires much less computation time than FIML. For a sample size of 1,000, the average number of seconds per sample computed across 50 trials was 793 seconds under FIML and 179 seconds under GMM. We conjecture that the ratio of computation time for GMM and FIML falls as the difficulty of the loss minimization problem increases. The control problem will be more difficult when the number of policy-rule coefficients is larger or when the optimality hypothesis is false.

B.2. Estimation accuracy

We next compare the accuracy of parameter estimates obtained under GMM and FIML

²Estimation was performed with Visual Fortran Professional Edition 6.0.A on an IBM desktop computer with an Intel Pentium 4 CPU rated at 1.6 GHz and 1.25 GB of RAM.

by conducting Monte Carlo experiments with two different assumption about policy. In the first case, the true policy-rule coefficients are optimal for a given loss function. In the second case, the true policy coefficients are not optimal for any loss function in the family we consider. For sample sizes of 100 and 1,000, Table 13 reports findings for the case in which the optimality hypothesis is true.³ At both sample sizes, FIML and GMM return unbiased estimates of the structural parameters, loss function weights, and policy-rule coefficients that converge to the true values with sample size. At sample sizes of 100, FIML estimation results in smaller standard errors in general and especially for the loss function weights. For sample sizes of 1,000, however, GMM estimation results in smaller bias and smaller standard errors for several key parameters and comparable bias and standard errors for the others.

Table 14 reports findings for the case in which the optimality hypothesis is false.⁴ Under FIML and GMM, imposing optimal policy restrictions when they are false generates bias in the estimates of some structural parameters. For example, the GMM estimate of λ is too small, indicating a reduced role for expected future output in the IS equation. The FIML estimate of λ is accurate but the estimate of a_1 is too low, pointing to a smaller impact of lagged output. Both procedures bias the estimate of β , the slope coefficient in the Phillips curve, towards zero. Both procedures also return biased estimates of the loss function weights that imply almost no concern for stabilizing the output gap or the nominal interest rate.

There is one major difference in the outcome of the two procedures when false optimality restrictions are imposed. GMM yields unbiased and efficient estimates of the policy-rule coefficients at all sample sizes. In contrast, the FIML estimates of the policy coefficients are significantly biased. The standard errors for the GMM estimates range between 0.02 and 0.08 when the sample size is 1,000. The FIML standard errors, however, are much larger, ranging between 0.25 and 0.70. Thus, a researcher using GMM could be confident that

³The parameterization of the forward-looking model is the same one described in Table 3 of section 3.2.

⁴The parameterization of the forward-looking model is the same one described in Table 4 of section 3.2.

policy-rule coefficient estimates were accurate regardless of whether or not the optimality hypothesis were true. The researcher using FIML could not be.

B.3. Further discussion of GMM results

Here we explain why GMM, in contrast to other estimators, consistently returns unbiased estimates of policy-rule coefficients but biased estimates of some structural parameters when the optimal policy hypothesis is false. To guide us in this part of the analysis, we estimated the forward-looking model subject to the optimal policy restrictions for a synthetic data set with 10,000 observations. After obtaining parameter estimates, we recorded the (16×16) optimal weighting matrix and the (16×13) matrix of partial derivatives of the GMM sample moments with respect to the estimated parameter vector. This exercise was conducted for the case where the optimal policy restriction was true and also for the case where it was false. A comparison of the output from both cases revealed several relevant points.

First, the (4×4) partition of the GMM weighting matrix corresponding to the partial derivatives of expected loss has elements that are large relative to elements outside the partition. For the case in which the optimality hypothesis is true, the diagonal elements of the (4×4) partition are on the order of 10^5 while the elements outside the partition are on the order of 10^1 . This divergence occurs because variation in the partial derivatives across sample observations is much smaller than variation in the correlation between the residuals and the regressors. For the case in which the optimality hypothesis is false, the ratio of diagonal elements inside and outside the partition is even larger. Because the optimal weighting matrix places considerable weight on the sample moments associated with the partial derivatives of expected loss, GMM essentially “tries harder” during the second iteration to set the partial derivatives to zero than to satisfy the least-squares normal equations. It does so by locating false values of the structural parameters that make the observed policy embedded in the reaction function coefficients appear optimal.

Second, the (16×13) matrix of partial derivatives of the GMM sample moments with respect to the estimated parameter vector reveals that each parameter has a sizeable marginal effect on the moment conditions when the optimality hypothesis is true. Small adjustments in the parameters appearing in the IS equation and the Phillips curve, for instance, have a large impact on the partial derivatives of expected loss and on the correlations involving the output and inflation residuals. The situation is very different when the optimality hypothesis is false. For example, the partial derivatives of the residual correlation moments with respect to λ and β are zero. Meanwhile, the marginal effect of both parameters on the partial derivatives of expected loss is substantial. In contrast, marginal changes in all of the other parameters of the model affect both sets of moment conditions simultaneously. It is as if GMM has assigned λ and β only one task—to satisfy the policymaker’s first order conditions—and has discovered a parameterization that minimizes the cost of this assignment. Not surprisingly, these are precisely the two parameters which display the most severe asymptotic bias.

Appendix C. Other issues relating to GMM

In this section we address two issues concerning the implementation of the GMM strategy introduced in Givens and Salemi (2007). First, the observation that policy-rule coefficients are always estimated without bias suggests that it may be desirable to perform estimation in two steps. In the first step, estimates of the structural parameters and policy-rule coefficients can be obtained from the least-squares normal equations alone. The loss function weights can then be retrieved from the information contained in the policymaker’s first order conditions while holding the first step estimates fixed. Second, it is common knowledge that estimation of economic models like the ones examined here are often sensitive to starting values. Thus, we check the robustness of our Monte Carlo results to variation in starting values.

C.1. A comparison of GMM and a two step procedure

We compare the performance of our unified approach to estimation with GMM to a two

step procedure used often in econometric policy evaluation. In the first step, structural parameters and policy-rule coefficients are estimated simultaneously and without imposing the optimal-policy restrictions. In the second step, the loss function weights are estimated while holding the first step estimates fixed. To implement the second step, we search over loss function weights for values that minimize a GMM criterion based solely on the partial derivatives of expected loss. Our belief is that the two step procedure should not perform as well as joint estimation with GMM when the hypothesis of policy optimality is true because the first-order conditions carry information that aids in the estimation of the model's structural parameters.

Table 15 shows that both procedures yield unbiased estimates of all parameters including the loss function weights, although the standard errors tend to be smaller when the optimal policy restrictions are imposed during estimation. The improvement in precision, even in samples as large as 10,000, comes from the additional information contained in the partial derivatives.

Table 16 reports a comparison of the partial derivative estimates for the unified and two step procedures and provides additional evidence that the former generates a more accurate picture of the data. It is clear that a two step approach would lead the researcher to the erroneous conclusion that the policymaker's first order conditions were not satisfied. At a sample size of 1,000, the mean absolute value of partial derivative estimates computed across policy-rule coefficients exceeds 100. Only at a sample size of 10,000 do the two step estimates of the partial derivatives approach zero. Consequently, a test of the model's over-identifying restrictions using the two-step estimates is very likely to reject the optimality hypothesis when it is true. Suppose we test the policy-optimality hypothesis using the GMM Q statistic obtained in the second stage of estimation. Under the null hypothesis, $T \times Q$ is distributed chi-squared with two degrees of freedom since there are four partial derivatives and two estimated loss function weights. For the samples reported in Table 16, the optimality

hypothesis would be rejected in 91 percent of samples of size 100, 99 percent of samples of size 1,000, and 100 percent of samples of size 10,000. As Table 5 of the paper makes clear, however, rejection rates for the over-identifying restrictions of the forward-looking model are much smaller when true optimal policy restrictions are imposed during GMM estimation.

C.2. Robustness to starting values

In this section we document the robustness of our findings to changes in parameter starting values. To examine how sensitive the results are to different initial values, we conduct a Monte Carlo experiment in which the structural parameters are reset to values either ten percent below or ten percent above the original starting values. We nest starting-value changes and draw two samples each of 64 different starting value combinations. For each sample, we set the starting values of the policy-rule coefficients to 0.01 and the starting values of the loss function weights to 1.0 so that each stabilization objective was equally weighted at the outset. We varied the sample size between 100 and 5,000.

We report our findings in Table 17. To facilitate comparisons, we display some values from Table 3 which reports findings based on our original starting values. The results clearly demonstrate that our estimates are robust to variation in starting values. Indeed, the results in the left and right panels are very similar in small samples and nearly identical in large samples.

We do not claim that the GMM procedure works well for arbitrary starting values. In particular, it is essential that the researcher choose starting values that satisfy the saddlepath restriction discussed by Blanchard and Kahn (1980) and others. If those values imply too many unstable roots, it is difficult for the algorithm to locate the stable region of the parameter space. If they imply too few unstable roots, it is sometimes difficult for the algorithm to locate the region of the parameter space where a unique stable solution resides.

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Table 1. Backward-looking model

(ρ, θ, W)	True Value	Optimality Restriction (True): Not Imposed				Optimality Restriction (True): Imposed			
		<i>sample size</i>				<i>sample size</i>			
		100	250	500	5000	100	250	500	5000
a	0.90	0.916 (.23)	0.890 (.04)	0.898 (.03)	0.900 (.01)	0.918 (.19)	0.892 (.04)	0.899 (.03)	0.900 (.01)
b	0.15	0.349 (.77)	0.163 (.13)	0.169 (.09)	0.150 (.03)	0.269 (.53)	0.171 (.12)	0.179 (.09)	0.150 (.03)
α	0.50	0.495 (.09)	0.493 (.06)	0.494 (.04)	0.500 (.01)	0.497 (.09)	0.493 (.06)	0.496 (.04)	0.500 (.01)
β	0.10	0.101 (.08)	0.105 (.05)	0.106 (.04)	0.099 (.01)	0.087 (.07)	0.107 (.06)	0.106 (.04)	0.099 (.01)
W_y	0.10	–	–	–	–	0.066 (.24)	0.131 (.23)	0.129 (.14)	0.111 (.05)
W_r	0.30	–	–	–	–	0.181 (.33)	0.452 (.48)	0.444 (.41)	0.320 (.12)
θ_y	0.306	0.291 (.09)	0.306 (.05)	0.306 (.03)	0.308 (.01)	0.296 (.08)	0.304 (.05)	0.305 (.03)	0.308 (.01)
θ_π	0.102	0.116 (.11)	0.097 (.07)	0.107 (.04)	0.101 (.01)	0.121 (.10)	0.110 (.06)	0.115 (.04)	0.101 (.01)
Q		.24e-2	.87e-4	.62e-5	.13e-17	.64e-2	.22e-2	.69e-3	.17e-8
Fraction		1.00	1.00	1.00	1.00	0.83	0.85	0.95	1.00

1. For the case in which the hypothesis of policy optimality is **true**, the table reports estimates of the following model: $y_t = ay_{t-1} - b(r_t - \pi_t) + u_t$, $\pi_t = \alpha\pi_{t-1} + \beta y_t + v_t$, $r_t = \theta_y y_{t-1} + \theta_\pi \pi_{t-1} + w_t$. The variables are defined as: y - output, π - inflation, r - interest rate. W_y and W_r are the loss function weights for y and r normalized by the unit weight attached to π .

2. Q is the GMM estimation criterion and Fraction reports the fraction of trials that result in no outliers.

3. The parenthesis contain standard errors computed across Fraction \times 100 trials for each sample size.

Table 2. Backward-looking model

(ρ, θ, W)	True Value	Optimality Restriction (False): Imposed <i>sample size</i>			
		100	250	500	5000
		a	0.90	0.875 (.06)	0.886 (.04)
b	0.15	0.138 (.06)	0.144 (.03)	0.141 (.02)	0.146 (.01)
α	0.50	0.485 (.09)	0.483 (.06)	0.485 (.04)	0.487 (.01)
β	0.10	0.047 (.05)	0.035 (.02)	0.030 (.01)	0.026 (.004)
W_y	0.10	0.4e-6 (.3e-5)	0.67e-6 (.6e-5)	0.17e-17 (.2e-17)	0.9e-18 (.11e-17)
W_r	0.30	0.003 (.002)	0.25e-2 (.07)	0.22e-2 (.001)	0.2e-2 (.01)
θ_y	0.20	0.177 (.07)	0.182 (.05)	0.184 (.03)	0.186 (.01)
θ_π	2.00	2.02 (.12)	2.00 (.07)	2.01 (.04)	2.01 (.01)
Q		0.020	0.017	0.016	0.014
Fraction		0.83	0.85	0.95	1.00

1. For the case in which the hypothesis of policy optimality is **false** and **imposed**, the table reports estimates of the following model: $y_t = ay_{t-1} - b(r_t - \pi_t) + u_t$, $\pi_t = \alpha\pi_{t-1} + \beta y_t + v_t$, $r_t = \theta_y y_{t-1} + \theta_\pi \pi_{t-1} + w_t$. The variables are defined as: y - output, π - inflation, r - interest rate. W_y and W_r are the loss function weights for y and r normalized by the unit weight attached to π .

2. Q is the GMM estimation criterion and Fraction reports the fraction of trials that result in no outliers.

3. The parenthesis contain standard errors computed across Fraction \times 100 trials for each sample size.

Table 3. Forward-looking model

(ρ, θ, W)	True Value	Optimality Restriction (True): Not Imposed				Optimality Restriction (True): Imposed			
		<i>sample size</i>				<i>sample size</i>			
		100	250	500	5000	100	250	500	5000
λ	0.15	0.207 (.25)	0.218 (.22)	0.181 (.19)	0.134 (.11)	0.186 (.23)	0.162 (.18)	0.139 (.15)	0.110 (.07)
a_1	1.10	1.04 (.26)	1.04 (.22)	1.07 (.19)	1.12 (.11)	1.02 (.23)	1.05 (.17)	1.08 (.14)	1.14 (.07)
a_2	-0.30	-0.279 (.12)	-0.271 (.08)	-0.290 (.07)	-0.303 (.04)	-0.301 (.12)	-0.300 (.08)	-0.316 (.07)	-0.307 (.02)
b	0.20	0.184 (.16)	0.184 (.12)	0.185 (.09)	0.209 (.05)	0.147 (.14)	0.164 (.10)	0.168 (.08)	0.219 (.04)
α_1	0.50	0.433 (.32)	0.407 (.24)	0.430 (.21)	0.507 (.06)	0.372 (.32)	0.349 (.23)	0.378 (.23)	0.504 (.06)
α_2	0.45	1.67 (12.0)	0.481 (.07)	0.469 (.06)	0.449 (.02)	1.04 (4.9)	0.472 (.07)	0.468 (.06)	0.448 (.01)
β	0.15	0.196 (.11)	0.187 (.08)	0.180 (.06)	0.150 (.02)	0.185 (.12)	0.191 (.09)	0.184 (.07)	0.150 (.01)
W_y	0.10	–	–	–	–	1.37 (8.1)	0.106 (.22)	0.587 (3.6)	0.076 (.07)
W_r	0.30	–	–	–	–	0.749 (6.3)	0.209 (.21)	0.228 (.19)	0.314 (.06)
θ_{y1}	1.10	1.09 (.13)	1.09 (.09)	1.09 (.06)	1.10 (.02)	1.08 (.15)	1.09 (.11)	1.09 (.07)	1.10 (.02)
θ_π	0.63	0.628 (.10)	0.610 (.07)	0.625 (.04)	0.627 (.02)	0.646 (.11)	0.635 (.07)	0.642 (.04)	0.627 (.01)
θ_r	0.23	0.238 (.08)	0.237 (.05)	0.236 (.04)	0.228 (.01)	0.246 (.08)	0.240 (.04)	0.238 (.04)	0.227 (.01)
θ_{y2}	-0.20	-0.193 (.19)	-0.189 (.11)	-0.196 (.07)	-0.197 (.02)	-0.209 (.18)	-0.197 (.11)	-0.205 (.07)	-0.197 (.02)
Q		0.032 (.04)	.75e-2 (.76e-2)	.41e-2 (.45e-2)	.22e-3 (.27e-3)	0.110 (.17)	0.036 (.04)	0.029 (.05)	.18e-2 (.60e-2)
Fraction		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

1. For the case in which the hypothesis of policy optimality is **true**, the table reports estimates of the following model: $y_t = \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + u_t$, $\pi_t = \beta y_t + \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + v_t$, $r_t = \theta_{y1} y_{t-1} + \theta_\pi \pi_{t-1} + \theta_r r_{t-1} + \theta_{y2} y_{t-2} + w_t$. The variables are defined as: y - output, π - inflation, r - interest rate. W_y and W_r are the loss function weights for y and r normalized by the unit weight attached to π .

2. Q is the GMM estimation criterion and Fraction reports the fraction of trials that result in no outliers.

3. The parenthesis contain standard errors computed across Fraction \times 100 trials for each sample size.

Table 4. Forward-looking model

(ρ, θ, W)	True Value	Optimality Restriction (False): Imposed			
		<i>sample size</i>			
		100	250	500	5000
λ	0.15	0.107 (.18)	0.092 (.13)	0.075 (.11)	0.027 (.03)
a_1	1.10	1.09 (.19)	1.15 (.12)	1.15 (.11)	1.20 (.03)
a_2	-0.30	-0.354 (.13)	-0.363 (.10)	-0.361 (.06)	-0.367 (.02)
b	0.20	0.181 (.11)	0.202 (.08)	0.206 (.06)	0.239 (.02)
α_1	0.50	0.568 (.27)	0.555 (.13)	0.559 (.13)	0.582 (.04)
α_2	0.45	0.372 (.10)	0.356 (.08)	0.351 (.05)	0.352 (.01)
β	0.15	0.119 (.06)	0.108 (.04)	0.111 (.03)	0.103 (.01)
W_y	0.10	0.003 (.02)	0.74e-4 (.74e-3)	0.78e-4 (.78e-3)	0.3e-17 (.4e-17)
W_r	0.30	0.001 (.004)	0.14e-3 (.07e-3)	0.46e-4 (.46e-2)	0.3e-17 (.5e-17)
θ_{y1}	0.50	0.489 (.11)	0.479 (.07)	0.468 (.06)	0.471 (.01)
θ_π	1.50	1.47 (.20)	1.52 (.09)	1.53 (.10)	1.54 (.02)
θ_r	0.50	0.512 (.06)	0.509 (.03)	0.506 (.04)	0.506 (.01)
θ_{y2}	0.00	0.009 (.18)	-0.82e-3 (.09)	0.003 (.10)	-0.006 (.02)
Q		0.224 (.23)	0.145 (.09)	0.141 (.11)	0.113 (.005)
Fraction		1.00	1.00	1.00	1.00

1. For the case in which the hypothesis of policy optimality is **false** and **imposed**, the table reports estimates of the following model: $y_t = \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + u_t$, $\pi_t = \beta y_t + \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + v_t$, $r_t = \theta_{y1} y_{t-1} + \theta_\pi \pi_{t-1} + \theta_r r_{t-1} + \theta_{y2} y_{t-2} + w_t$. The variables are defined as: y - output, π - inflation, r - interest rate. W_y and W_r are the loss function weights for y and r normalized by the unit weight attached to π .

2. Q is the GMM estimation criterion and Fraction reports the fraction of trials that result in no outliers.

3. The parenthesis contain standard errors computed across Fraction \times 100 trials for each sample size.

Table 5. Rejection frequency of over-identifying restrictions

Optimal Policy Restriction		Degrees of Freedom	Sample Size	Test Size				
				.25	.10	.05	.025	.01
True	Not Imposed	1	100	60	40	29	17	11
			250	51	27	12	6	2
			500	52	28	17	10	5
			5000	32	13	5	2	1
True	Imposed	3	100	64	48	40	34	24
			250	63	43	39	32	28
			500	60	48	42	38	29
			5000	41	28	20	18	14
False	Imposed	3	100	98	96	96	91	81
			250	100	100	100	100	100
			500	100	100	100	100	100
			5000	100	100	100	100	100

Note: For the forward-looking model, the table reports the frequency of rejection of the over-identifying moment restrictions as a function of test size, sample size, whether or not the optimality restriction is true, and whether or not the optimality restriction is imposed during estimation. The values recorded are given in percentages and are computed across 100 trials for each sample size.

Table 6. Parameters for the representative agent model

Parameter	Description	Value
b	degree of habit formation	0.65
σ	inverse of the intertemporal elasticity of substitution	2.00
γ	degree of partial price indexation	0.75
β	household subjective discount factor	0.99*
ε	fraction of firms unable to reset prices	0.50
χ	inverse of the wage elasticity of labor supply	2.00
α	capital elasticity of output	0.33*
η	elasticity of demand for intermediate goods	11.0*
θ_π	optimal policy rule coefficient on inflation	9.28
θ_y	optimal policy rule coefficient on output	0.28
θ_r	optimal policy rule coefficient on the interest rate	1.63
W_π	implied preference weight on inflation objective	10.9**
W_y	implied preference weight on output gap objective	10.6**
δ_y	implied strength of the lag in output gap objective	0.49**

Note: * - indicates that the parameter is fixed at the given value during estimation; ** - indicates a parameter value that is implied by the values of the other parameters.

Table 7. Representative agent model

(ρ, θ, W)	True Value	Optimality Restriction (True): Not Imposed				Optimality Restriction (True): Imposed			
		<i>sample size</i>				<i>sample size</i>			
		100	250	500	5000	100	250	500	5000
b	0.65	0.702 (.21)	0.670 (.17)	0.669 (.11)	0.650 (.03)	0.739 (.23)	0.698 (.18)	0.688 (.12)	0.650 (.03)
σ	2.00	3.34 (4.7)	2.75 (2.4)	2.19 (1.4)	2.04 (.48)	3.32 (5.9)	2.28 (1.9)	1.98 (1.3)	2.02 (.45)
γ	0.75	0.812 (.79)	1.08 (1.3)	0.831 (.41)	0.761 (.13)	0.740 (.41)	0.747 (.23)	0.741 (.14)	0.763 (.01)
ε	0.50	0.517 (.10)	0.483 (.09)	0.492 (.06)	0.500 (.02)	0.502 (.04)	0.500 (.02)	0.500 (.01)	0.498 (.002)
χ	2.00	2.29 (2.0)	1.87 (1.2)	1.90 (.80)	2.01 (.27)	2.13 (1.1)	1.99 (.58)	2.04 (.36)	1.98 (.12)
W_y	10.6	–	–	–	–	12.4 (7.8)	10.7 (2.3)	10.7 (1.6)	10.5 (.48)
W_π	10.9	–	–	–	–	11.5 (3.1)	11.1 (1.9)	10.9 (.95)	10.8 (.12)
θ_y	0.28	0.281 (.06)	0.278 (.04)	0.277 (.03)	0.277 (.01)	0.276 (.06)	0.276 (.04)	0.277 (.03)	0.277 (.01)
θ_π	9.28	9.28 (.11)	9.29 (.07)	9.28 (.04)	9.28 (.02)	9.31 (.15)	9.29 (.08)	9.28 (.05)	9.28 (.02)
θ_r	1.63	1.63 (.02)	1.63 (.02)	1.63 (.01)	1.63 (.003)	1.64 (.03)	1.64 (.02)	1.63 (.01)	1.63 (.003)
Q		0.013 (.01)	0.004 (.006)	0.002 (.003)	.14e-3 (.17e-3)	0.083 (.11)	0.036 (.06)	0.015 (.04)	.78e-3 (.59e-3)
Fraction		0.88	0.99	1.00	1.00	0.88	0.99	1.00	1.00

1. For the case in which the hypothesis of policy optimality is **true**, the table reports estimates of the representative agent model described in section 3.3. The parameters have the following interpretation: b - habit persistence, σ - inverse of the intertemporal elasticity of substitution, γ - partial indexation, ε - fraction of firms unable to adjust prices, χ - inverse of the wage elasticity of labor supply. $\{\theta_y, \theta_\pi, \theta_r\}$ are the coefficients of the policy rule and $\{W_y, W_\pi\}$ are the loss function weights.

2. Q is the GMM estimation criterion and Fraction reports the fraction of trials that result in no outliers.

3. The parenthesis contain standard errors computed across Fraction \times 100 trials for each sample size.

Table 8. Representative agent model

(ρ, θ, W)	True Value	Optimality Restriction (False):			
		Imposed			
		<i>sample size</i>			
		100	250	500	5000
b	0.65	0.749 (.23)	0.739 (.21)	0.786 (.17)	0.768 (.11)
σ	2.00	2.77 (4.4)	3.14 (5.6)	1.34 (2.2)	0.717 (.56)
γ	0.75	0.937 (.51)	0.987 (.41)	0.966 (.32)	0.995 (.05)
ε	0.50	0.396 (.12)	0.368 (.08)	0.357 (.07)	0.342 (.01)
χ	2.00	1.12 (.66)	1.08 (.48)	1.20 (.39)	1.28 (.17)
W_y	10.6	10.2 (7.2)	10.6 (10.0)	7.83 (3.7)	6.82 (.61)
W_π	10.9	8.49 (9.4)	5.98 (4.8)	5.31 (3.7)	4.31 (.18)
θ_y	0.50	0.484 (.08)	0.494 (.05)	0.495 (.04)	0.499 (.01)
θ_π	1.50	1.53 (.05)	1.52 (.03)	1.52 (.02)	1.52 (.01)
θ_r	0.50	0.499 (.03)	0.502 (.02)	0.499 (.01)	0.496 (.01)
Q		0.279 (.14)	0.260 (.09)	0.259 (.07)	0.244 (.03)
Fraction		0.91	0.99	1.00	1.00

1. For the case in which the hypothesis of policy optimality is **false** and **imposed**, the table reports estimates of the representative agent model described in section 3.3. The parameters have the following interpretation: b - habit persistence, σ - inverse of the intertemporal elasticity of substitution, γ - partial indexation, ε - fraction of firms unable to adjust prices, χ - inverse of the wage elasticity of labor supply. $\{\theta_y, \theta_\pi, \theta_r\}$ are the coefficients of the policy rule and $\{W_y, W_\pi\}$ are the loss function weights.

2. Q is the GMM estimation criterion and Fraction reports the fraction of trials that result in no outliers.

3. The parenthesis contain standard errors computed across Fraction \times 100 trials for each sample size.

Table 9. Rejection frequency of over-identifying restrictions

Optimal Policy Restriction		Degrees of Freedom	Sample Size	Test Size				
				.25	.10	.05	.025	.01
True	Not Imposed	1	100	60	33	26	18	15
			250	23	10	6	5	2
			500	27	12	6	4	2
			5000	16	4	1	0	0
True	Imposed	4	100	36	25	19	16	16
			250	25	14	12	11	9
			500	28	9	6	4	4
			5000	19	11	6	3	2
False	Imposed	4	100	100	99	98	92	88
			250	100	100	100	100	100
			500	100	100	100	100	100
			5000	100	100	100	100	100

Note: For the representative agent model, the table reports the frequency of rejection of the over-identifying moment restrictions as a function of test size, sample size, whether or not the optimality restriction is true, and whether or not the optimality restriction is imposed during estimation. The values recorded are given in percentages and are computed across Fraction \times 100 trials for each sample size.

Table 10. Assessment of model fit (1979:III to 2001:IV)

	Backward Looking		Forward Looking		Representative Agent	
	Policy Restrictions:		Policy Restrictions:		Policy Restrictions:	
	<i>Not Imposed</i>	<i>Imposed</i>	<i>Not Imposed</i>	<i>Imposed</i>	<i>Not Imposed</i>	<i>Imposed</i>
Q	4.3e-19	0.016	0.018	0.094	0.033	1.108
$Q \times T$	–	–	1.55	8.27	2.94	97.5
p-value	–	–	0.15	0.02	0.09	0.00
\mathcal{L}	1219.1	1215.4	1274.7	1254.7	1266.4	1083.9

Note: Q is the minimized GMM estimation criterion. $Q \times T$ is the Hansen (1982) chi-squared test statistic for the model's over-identifying restrictions. $\mathcal{L} = -\frac{T}{2} \ln(|\Phi|)$ corresponds to pseudo log likelihood and is obtained from the residual-error covariance matrix Φ .

Table 11. Structural parameter estimates (1979:III to 2001:IV)

A. Forward-Looking Model		
Parameter	Optimal Policy Restrictions:	
	<i>Not Imposed</i>	<i>Imposed</i>
λ	0.42 (12.4)	0.08 (5.4)
a_1	0.77 (8.9)	1.18 (.42)
a_2	-0.20 (3.4)	-0.30 (.15)
b	0.01 (19.7)	0.01 (3.4)
α_1	0.00 (2.5e4)	0.02 (21.2)
α_2	0.67 (.16)	0.71 (.36)
β	0.04 (.74)	0.02 (1.2)
B. Representative Agent Model		
Parameter	Optimal Policy Restrictions:	
	<i>Not Imposed</i>	<i>Imposed</i>
b	0.87 (.80)	0.999 (4.9e3)
σ	4.43 (1.6)	0.47e-4 (5.0e3)
γ	0.76 (.65)	1.00 (1.1e4)
ε	0.98 (4.2)	0.82 (.32)
χ	0.02 (1.4e3)	0.9e-6 (4.9e3)

Note: The table reports structural parameter estimates of the forward-looking and representative agent models described in sections 3.2 and 3.3. The numbers in parenthesis are estimated standard errors.

Table 12. Policy coefficient estimates (1979:III to 2001:IV)

A. Forward-Looking Model				
Coefficient	Optimal Policy Restrictions:			
	<i>Not Imposed</i>		<i>Imposed</i>	
	Estimate	Derivative	Estimate	Derivative
θ_{y1}	0.21 (.18)	-34.4	0.59 (.18)	-0.9e-3
θ_{π}	0.28 (.11)	-9.8	0.29 (.09)	-0.2e-3
θ_r	0.78 (.09)	6.3	0.87 (.08)	-0.5e-4
θ_{y2}	-0.26 (.17)	-32.7	-0.13 (.17)	-0.8e-3
W_y	–	–	0.00 (2.7e5)	–
W_r	–	–	0.00 (2.5e5)	–

B. Representative Agent Model				
Coefficient	Optimal Policy Restrictions:			
	<i>Not Imposed</i>		<i>Imposed</i>	
	Estimate	Derivative	Estimate	Derivative
θ_y	-0.05 (.03)	-4.5e3	-1.48 (.96)	0.05
θ_{π}	0.31 (.11)	-7.9e2	6.69 (4.5)	-0.08
θ_r	0.80 (.08)	-17.3	-0.32e-3 (.56)	-0.28
W_y	123.1	–	20.6	–
W_{π}	1.3e4	–	139.0	–

Note: The table reports policy-rule coefficient estimates and loss function weights for the forward-looking and representative agent models described in sections 3.2 and 3.3. The numbers in parenthesis are estimated standard errors.

Table 13. Comparison of GMM and FIML

(ρ, θ, W)	True Value	Optimality Restriction (True):			
		FIML		GMM	
		<i>sample size</i>		<i>sample size</i>	
		100	1000	100	1000
λ	0.15	0.155 (.11)	0.174 (.08)	0.181 (.23)	0.114 (.11)
a_1	1.10	0.957 (.16)	0.977 (.11)	1.04 (.24)	1.12 (.11)
a_2	-0.30	-0.242 (.09)	-0.273 (.05)	-0.307 (.12)	-0.315 (.04)
b	0.20	0.125 (.08)	0.116 (.07)	0.152 (.15)	0.197 (.07)
α_1	0.50	0.453 (.12)	0.461 (.06)	0.372 (.32)	0.448 (.17)
α_2	0.45	0.412 (.07)	0.409 (.05)	0.749 (2.1)	0.454 (.03)
β	0.15	0.160 (.08)	0.167 (.04)	0.187 (.13)	0.164 (.04)
W_y	0.10	0.129 (.18)	0.110 (.07)	12.4 (101)	0.088 (.11)
W_r	0.30	0.127 (.17)	0.135 (.12)	0.609 (4.9)	0.283 (.15)
θ_{y1}	1.10	1.06 (.15)	1.07 (.07)	1.08 (.14)	1.10 (.04)
θ_π	0.63	0.682 (.11)	0.676 (.06)	0.649 (.11)	0.633 (.03)
θ_r	0.23	0.239 (.06)	0.242 (.03)	0.244 (.08)	0.233 (.02)
θ_{y2}	-0.20	-0.189 (.17)	-0.195 (.08)	-0.209 (.18)	-0.206 (.05)

Note: For the case in which the hypothesis of policy optimality is **true** and **imposed**, the table reports estimates of the structural parameters, loss function weights, and policy-rule coefficients for the forward-looking model described in section 3.2. The estimates in the left panel are obtained from a nested approach that uses full information maximum likelihood (FIML). The estimates in the right panel are obtained using GMM. The numbers in parenthesis are estimated standard errors.

Table 14. Comparison of GMM and FIML

(ρ, θ, W)	True Value	Optimality Restriction (False):			
		FIML		GMM	
		<i>sample size</i>		<i>sample size</i>	
		100	1000	100	1000
λ	0.15	0.151 (.08)	0.144 (.08)	0.107 (.18)	0.045 (.05)
a_1	1.10	0.943 (.10)	0.929 (.10)	1.09 (.19)	1.18 (.06)
a_2	-0.30	-0.196 (.08)	-0.187 (.10)	-0.354 (.13)	-0.372 (.03)
b	0.20	0.177 (.05)	0.183 (.05)	0.181 (.11)	0.222 (.03)
α_1	0.50	0.614 (.07)	0.629 (.04)	0.568 (.27)	0.562 (.08)
α_2	0.45	0.461 (.05)	0.445 (.05)	0.372 (.10)	0.347 (.04)
β	0.15	0.057 (.02)	0.070 (.03)	0.119 (.06)	0.108 (.02)
W_y	0.10	0.008 (.3e-2)	0.007 (.3e-2)	0.003 (.02)	0.7e-15 (.5e-14)
W_r	0.30	0.026 (.9e-2)	0.028 (.03)	0.001 (.4e-2)	0.55e-4 (.4e-3)
θ_{y1}	0.50	0.262 (.70)	0.318 (.52)	0.489 (.11)	0.479 (.03)
θ_π	1.50	-0.144 (.92)	-0.299 (.70)	1.47 (.20)	1.53 (.08)
θ_r	0.50	0.639 (.30)	0.649 (.25)	0.512 (.06)	0.510 (.02)
θ_{y2}	0.00	-0.259 (.64)	-0.312 (.53)	0.009 (.18)	-0.010 (.06)

Note: For the case in which the hypothesis of policy optimality is **false** and **imposed**, the table reports estimates of the structural parameters, loss function weights, and policy-rule coefficients for the forward-looking model described in section 3.2. The estimates in the left panel are obtained from a nested approach that uses full information maximum likelihood (FIML). The estimates in the right panel are obtained using GMM. The numbers in parenthesis are estimated standard errors.

Table 15. Two step estimation of the forward-looking model

(ρ, θ, W)	True Value	Two Step Procedure			Unified Estimation		
		<i>sample size</i>			<i>sample size</i>		
		100	1000	10000	100	1000	10000
λ	0.15	0.207 (.25)	0.157 (.17)	0.131 (.09)	0.181 (.23)	0.114 (.11)	0.112 (.05)
a_1	1.10	1.04 (.26)	1.09 (.17)	1.12 (.09)	1.04 (.24)	1.12 (.11)	1.13 (.06)
a_2	-0.30	-0.279 (.12)	-0.299 (.06)	-0.304 (.03)	-0.307 (.12)	-0.315 (.04)	-0.304 (.01)
b	0.20	0.184 (.16)	0.198 (.08)	0.209 (.04)	0.152 (.15)	0.197 (.07)	0.217 (.03)
α_1	0.50	0.433 (.32)	0.472 (.16)	0.506 (.04)	0.372 (.32)	0.448 (.17)	0.505 (.01)
α_2	0.45	1.67 (12.0)	0.459 (.04)	0.449 (.01)	0.749 (2.1)	0.454 (.03)	0.450 (.01)
β	0.15	0.196 (.11)	0.164 (.05)	0.149 (.01)	0.187 (.13)	0.164 (.04)	0.149 (.01)
W_y	0.10	0.679 (2.7)	0.120 (.25)	0.081 (.07)	12.4 (101)	0.088 (.11)	0.073 (.06)
W_r	0.30	5.2e20 (5.2e21)	0.222 (.14)	0.276 (.07)	0.609 (4.9)	0.283 (.15)	0.321 (.04)
θ_{y1}	1.10	1.09 (.13)	1.10 (.04)	1.10 (.01)	1.08 (.14)	1.10 (.04)	1.10 (.01)
θ_π	0.63	0.628 (.10)	0.621 (.03)	0.628 (.01)	0.649 (.11)	0.633 (.03)	0.628 (.01)
θ_r	0.23	0.238 (.08)	0.232 (.03)	0.228 (.01)	0.244 (.08)	0.233 (.02)	0.226 (.01)
θ_{y2}	-0.20	-0.193 (.19)	-0.199 (.05)	-0.198 (.01)	-0.209 (.18)	-0.206 (.05)	-0.196 (.02)

Note: The table reports estimates of the structural parameters, policy-rule coefficients, and loss function weights for the forward-looking model described in section 3.2. The figures in the left panel are obtained from a two step procedure whereby the normal equations alone are used to estimate structural parameters and policy-rule coefficients. The policymaker's first-order conditions are then used in a second step to estimate the loss function weights holding fixed the first step estimates. The figures in the right panel are obtained from the unified approach described in section 2. The numbers in parenthesis are standard errors.

Table 16. Two step estimation of the forward-looking model

Partial Derivative	Two Step Procedure			Unified Estimation		
	<i>sample size</i>			<i>sample size</i>		
	100	1000	10000	100	1000	10000
θ_{y1}	1.3e24 (1.3e25)	33.7 (216)	0.13e-4 (.6e-3)	-0.46e-3 (.3e-2)	-0.37e-5 (.2e-3)	0.21e-5 (.6e-5)
θ_{π}	1.9e24 (1.9e25)	-63.6 (408)	-0.8e-3 (.8e-3)	0.21e-3 (.6e-2)	-0.28e-4 (.1e-3)	0.33e-6 (.3e-5)
θ_r	3.3e25 (3.3e26)	-176 (1.1e4)	-0.13e-2 (.2e-2)	-0.53e-3 (.2e-2)	-0.58e-4 (.1e-3)	-0.35e-5 (.9e-5)
θ_{y2}	6.1e24 (6.1e25)	238 (1.6e4)	-0.17e-3 (.8e-3)	-0.35e-3 (.3e-2)	0.68e-5 (.2e-3)	0.16e-5 (.6e-5)

Note: The table reports estimates of the partial derivatives of loss with respect to the policy-rule coefficients of the forward-looking model described in section 3.2. The derivatives in the left panel are computed on the basis of a two step procedure. The derivatives in the right panel are computed on the basis of the unified approach to estimation. The numbers in parenthesis are standard errors.

Table 17. Estimation with alternative starting values

(ρ, θ, W)	True Value	Alternative Starting Values			Original Starting Values		
		<i>sample size</i>			<i>sample size</i>		
		100	500	5000	100	500	5000
λ	0.15	0.175 (.23)	0.142 (.15)	0.112 (.07)	0.186 (.23)	0.139 (.15)	0.110 (.07)
a_1	1.10	1.05 (.26)	1.08 (.15)	1.14 (.07)	1.02 (.23)	1.08 (.14)	1.14 (.07)
a_2	-0.30	-0.323 (.14)	-0.309 (.06)	-0.306 (.01)	-0.301 (.12)	-0.316 (.07)	-0.307 (.02)
b	0.20	0.147 (.13)	0.179 (.09)	0.222 (.04)	0.147 (.14)	0.168 (.08)	0.219 (.04)
α_1	0.50	0.368 (.31)	0.398 (.22)	0.507 (.02)	0.372 (.32)	0.378 (.23)	0.504 (.06)
α_2	0.45	0.469 (.10)	0.466 (.05)	0.449 (.01)	1.04 (4.9)	0.468 (.06)	0.448 (.01)
β	0.15	0.197 (.12)	0.181 (.06)	0.149 (.01)	0.185 (.12)	0.184 (.07)	0.150 (.01)
W_y	0.10	2.48 (18.3)	0.526 (3.6)	0.071 (.07)	1.37 (8.1)	0.587 (3.6)	0.076 (.07)
W_r	0.30	0.466 (3.2)	0.257 (.19)	0.326 (.06)	0.749 (6.3)	0.228 (.19)	0.314 (.06)
θ_{y1}	1.10	1.09 (.14)	1.09 (.06)	1.10 (.01)	1.08 (.15)	1.09 (.07)	1.10 (.02)
θ_π	0.63	0.654 (.11)	0.639 (.04)	0.627 (.01)	0.646 (.11)	0.642 (.04)	0.627 (.01)
θ_r	0.23	0.243 (.08)	0.234 (.03)	0.226 (.01)	0.246 (.08)	0.238 (.04)	0.227 (.01)
θ_{y2}	-0.20	-0.212 (.18)	-0.201 (.07)	-0.197 (.02)	-0.209 (.18)	-0.205 (.07)	-0.197 (.02)
Q		0.104 (.18)	0.021 (.04)	0.97e-3 (.1e-2)	0.110 (.17)	0.029 (.05)	0.18e-2 (.60e-2)

Note: The table reports estimates of the structural parameters, loss function weights, and policy-rule coefficients for the forward-looking model described in section 3.2. The estimates in the right panel are obtained conditional on the original parameter starting values. The estimates in the left panel are obtained conditional on random parameter starting values either 10 percent below or 10 percent above the original values. The numbers in parenthesis are standard errors.