Appendix to "A Note on Comparing Deep and Aggregate Habit Formation in an Estimated New-Keynesian Model"

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Appendix A. The Symmetric Equilibrium

A.1. Nonlinear Equilibrium Conditions

$$a_t/(x_t - b^a x_{t-1}) = \lambda_t \tag{A.1}$$

$$h_t^{\chi}(x_t - b^a x_{t-1}) = w_t \tag{A.2}$$

$$\lambda_t = \beta R_t E_t(\lambda_{t+1}/\pi_{t+1}) \tag{A.3}$$

$$x_t = c_t - b^d c_{t-1} \tag{A.4}$$

$$w_t = \gamma_t z_t \tag{A.5}$$

$$\nu_t = 1 - \gamma_t + \beta b^d E_t(\lambda_{t+1}/\lambda_t)\nu_{t+1} \tag{A.6}$$

$$c_t = \eta \nu_t x_t + \alpha(\pi_t/\pi - 1)(\pi_t/\pi)y_t - \alpha\beta E_t(\lambda_{t+1}/\lambda_t)(\pi_{t+1}/\pi - 1)(\pi_{t+1}/\pi)y_{t+1} \tag{A.7}$$

$$y_t = z_t h_t \tag{A.8}$$

$$y_t = c_t + (\alpha/2)(\pi_t/\pi - 1)^2 y_t \tag{A.9}$$

$$\log(R_t/R) = \theta_r \log(R_{t-1}/R) + (1-\theta_r)\left[\theta_\pi \log(\pi_t/\pi) + \theta_y \log(y_t/y)\right] + \varepsilon_{r,t} \tag{A.10}$$

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t} \tag{A.11}$$

$$\log z_t = (1-\rho_z)\log z + \rho_z \log z_{t-1} + \varepsilon_{z,t} \tag{A.12}$$

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A.2. Log-linear Approximation

$$\hat{a}_t - (\hat{x}_t - b^a \hat{x}_{t-1})/(1-b^a) = \hat{\lambda}_t \tag{A.13}$$

$$\chi \hat{h}_t + (\hat{x}_t - b^a \hat{x}_{t-1})/(1-b^a) = \hat{w}_t \tag{A.14}$$

$$\hat{\lambda}_t = \hat{R}_t + E_t \hat{\lambda}_{t+1} - E_t \hat{\pi}_{t+1} \tag{A.15}$$

$$\hat{x}_t = \left(\hat{c}_t - b^d \hat{c}_{t-1}\right)/(1-b^d) \tag{A.16}$$

$$\hat{w}_t = \hat{\gamma}_t + \hat{z}_t \tag{A.17}$$

$$\hat{\nu}_t = -\left(\eta(1-b^d) - (1-\beta b^d)\right)\hat{\gamma}_t + \beta b^d E_t \left(\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{\nu}_{t+1}\right) \tag{A.18}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + (1/\alpha)(\hat{c}_t - \hat{\nu}_t - \hat{x}_t) \tag{A.19}$$

$$\hat{y}_t = \hat{z}_t + \hat{h}_t \tag{A.20}$$

$$\hat{y}_t = \hat{c}_t \tag{A.21}$$

$$\hat{R}_t = \theta_r \hat{R}_{t-1} + (1-\theta_r)\left[\theta_\pi \hat{\pi}_t + \theta_y \hat{y}_t\right] + \varepsilon_{r,t} \tag{A.22}$$

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a,t} \tag{A.23}$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t} \tag{A.24}$$

Appendix B. Identification of Consumption Habits

The standard errors associated with b^d and b^a reported in Table 1 of the paper indicate that estimates of the deep habits parameter are far more precise than estimates of the aggregate habits parameter. To gain further insight into how well the time-series data on consumption, inflation, and the nominal interest rate identify estimates of both parameters in the nested habits model, I construct a surface plot of the log likelihood function in (b^d, b^a) space. All of the other model parameters are held fixed at their maximum likelihood estimates.

Fig. B.1 of the appendix confirms that deep habits impose restrictions on the data that enable sharp estimation of b^d. For values of b^d just below the point estimate of 0.9438, log likelihood declines rapidly regardless of the particular value of b^a. Moreover, for values of b^d above 0.96, the nested habits model becomes indeterminate. A different story emerges when one considers cross sections of log likelihood for any fixed value of b^d. In these cases log likelihood appears relatively flat for values of b^a anywhere between 0.50 and 0.70. A close inspection of the likelihood surface, however, reveals that there is enough curvature to identify an estimate of b^a, albeit with less precision than that of b^d.

Appendix C. The Degree of Price Stickiness

C.1. Are the Results Robust to 3-Quarter Price Contracts?

Due to difficulties in estimating the price adjustment cost parameter, the results reported in Table 1 are obtained by fixing α at a value that renders the aggregate price-setting equation identical to a Calvo-Yun Phillips curve in which prices are reset once every four quarters on average. Although price contracts lasting one year are common in the new-Keynesian literature, there is some compelling micro-level evidence suggesting that prices change more often. In this section I examine the robustness of the main empirical results to changes in the degree of price stickiness. Specifically, I re-estimate all three models under the assumption that $\alpha = 29.7919$, which is equivalent to a price change frequency of three quarters in a Calvo-Yun framework. The findings are reported in Table C.1 below.

Most of the key results are robust to lowering the degree of price stickiness. The deep habits model still outperforms the aggregate habits model in terms of overall fit as measured by log likelihood. In both cases, however, the maximized value of log likelihood is smaller than the corresponding values in Table 1. This suggests that the data strictly prefers more price rigidity to less regardless of the habit specification. The point estimates of the deep and aggregate habits models are also quite similar to the benchmark estimates in Table 1, indicating that uncertainty about the appropriate degree of price stickiness may not have serious implications for parameter inference. Finally, estimates of the nested habits model appear to be somewhat more sensitive to the particular value of α. It turns out that with three-quarter price contracts, the nested habits model is no longer significantly different from the deep habits model since the point estimate of b^a is arbitrarily close to zero.

C.2. Does the Deep Habits Model Require Significant Price Stickiness?

A central finding of the paper is that the intertemporal and price-elasticity effects introduced by deep habits combine with exogenous nominal rigidities to produce a model capable of generating significant inflation inertia. Since both of these effects vanish in the aggregate habits model, a natural question is whether the deep habits model still compares favorably to the aggregate habits model when the amount of exogenous price stickiness is lowered in the former but held constant in the latter. I address this question here by re-estimating the deep habits model several times each with progressively smaller amounts of price stickiness.

I consider values of the adjustment cost parameter α that imply price contracts lasting from as much as one year to as little as four months. The estimates are reported in Table C.2.

Estimates of the deep habits model are remarkably robust to a wide range of values for α. The estimates of b^d are always above 0.90 and the policy rule coefficients are quite stable. The Frisch labor supply elasticity χ declines rapidly as the degree of price stickiness falls, but none of these point estimates turn out to be statistically significant. The maximized value of log likelihood drops from 2377.65 to 2372.55 as price contracts shorten from one year to four months. Even under four-month contracts, however, log likelihood is still much higher under deep habits than it is with one-year price contracts under aggregate habits. This suggests that the *endogenous* price rigidities imparted by deep habits play a key role in improving model fit irrespective of the degree of *exogenous* price rigidities.

Appendix D. Autocorrelated Preference Shocks

D.1. What Explains the Distribution of Preference Shocks in the Nested Habits Model?

The lack of persistence and high volatility of preference shocks in the nested habits model is somewhat atypical of most DSGE models. To provide some intuition for why it appears, I consider a log-linear approximation of the consumption Euler equation given by

$$\hat{c}_t = \left(\frac{b^a + b^d + b^a b^d}{1 + b^a + b^d}\right)\hat{c}_{t-1} - \left(\frac{b^a b^d}{1 + b^a + b^d}\right)\hat{c}_{t-2} + \left(\frac{1}{1 + b^a + b^d}\right)E_t\hat{c}_{t+1} - \left(\frac{(1-b^a)(1-b^d)}{1 + b^a + b^d}\right)\left(\hat{R}_t - E_t\hat{\pi}_{t+1} - (1-\rho_a)\hat{a}_t\right), \tag{D.1}$$

where $\hat{X}_t \equiv \log X_t - \log X$ denotes the log deviation of a variable X_t from steady state X.[1] It is clear that positive values of b^a and b^d lower the impact effect of a given realization of $\hat{a}_t$ on consumption $\hat{c}_t$. Explaining the historical variation in US consumption data therefore requires larger innovations to the preference shock, all else equal, than would be necessary in the absence of deep or aggregate habit formation (see Table 1 in the paper). The autocorrelation coefficient ρ_a plays a similar role in the transmission of preference shocks, but its estimate is also likely being influenced by the presence of two consumption lags in the Euler equation. When either habit type is dropped from the utility function ($b^d = 0$ or $b^a = 0$), the second lag vanishes, forcing the model to rely more heavily on persistent shocks rather than its own internal structure to replicate the time-series properties of aggregate consumption.

[1] Eq. (D.1) can be derived by combining Eqs. (A.13), (A.15), and (A.16).

D.2. What Happens when Preference Shocks are Restricted to be Positively Autocorrelated?

Many empirical DSGE models that feature exogenous variation to households' time rate of preference find that these shocks tend to be positively serially correlated. At the same time, the models usually incorporate consumption habits only at the level of finished goods. In this section I re-estimate the nested habits model under the restriction that $\rho_a \in [0, 1)$. The goal here is to examine how the estimates change when a prior of no negative serial correlation is imposed on the parameter space. The results are displayed in Table D.1 below.

Forcing ρ_a to live on the unit interval drives the estimate of b^a to zero, the estimate of ρ_a to 0.5011, and the estimate of σ_a to 0.1229, which causes the model to be indistinguishable from the deep habits specification. This also lowers maximized log likelihood from 2380.41 to 2377.65. Thus when viewing the data through the lens of a nested habits model, the most likely interpretation of that data is one in which households have moderate consumption habits over the aggregate finished good, strong habits over differentiated products, and slightly negative serial correlation in the preference shocks. Requiring the serial correlation to be positive *a priori*, worsens model fit and conceals any evidence of aggregate habits.

Appendix E. Additional Model Simulations

To isolate the role of deep habits, the paper compares simulation results from the estimated deep habits model (second column of Table 1) to an identically-parameterized aggregate habits model. The ensuing differences in model dynamics are therefore driven entirely by the habit mechanism and not by variation in the parameter estimates. Although this type of comparison is necessary for proper identification, it discards the point estimates of both the nested and aggregate habits models, both of which provide information about the relative likelihoods of the competing models. As a result, in this section I report standard deviations and autocorrelations of detrended consumption, inflation, and the nominal interest rate from the estimated nested habits model (first column of Table 1) and the estimated aggregate habits model (third column of Table 1). The results along with the corresponding set of moments generated from a VAR(4) are reported in Table E.1 and Fig. E.1.

E.1. The Estimated Nested Habits Model

The standard deviations generated from the nested habits model are all close to the point estimates taken from a VAR(4) and easily within the 90% confidence intervals. Differences between the two are therefore insignificant at the 10% level. This result is perhaps not surprising given that the estimated deep habits model was already shown to account well for the joint volatility of $\hat{c}_t$, $\hat{\pi}_t$, and $\hat{R}_t$. The autocorrelation function from the nested model also matches closely some of the key autocorrelations in the data. In particular, the own correlations of inflation and the nominal interest rate both exhibit substantial persistence in line with the VAR evidence.

E.2. The Estimated Aggregate Habits Model

The estimated aggregate habits model does much worse in matching many of the same second moments taken from the data. For example, the standard deviation of detrended consumption is 0.1950, which is considerably higher than the upper bound of the 90% confidence interval for the corresponding VAR estimate. Moreover, Fig. E.1 shows that all of the correlations involving leads or lags of consumption are significantly higher than the correlations from the data. The reason why the aggregate habits model tends to overstate the volatility of (correlations involving) consumption is that by restricting $b^d = 0$, maximum likelihood drives up the estimates of ρ_a and ρ_z to 0.9478 and 0.9965, respectively (see third column of Table 1). While greater serial correlation in preference and technology shocks permits the aggregate habits model to better replicate the volatility and persistence of inflation and the nominal interest rate, it seriously undermines the model's ability to correctly identify consumption dynamics.

Table C.1

Parameter Estimates (1965:Q3 - 2012:Q1)

Model Parameter	Parameter Description	Nested Habits	Deep Habits	Aggregate Habits
σ_a	*preference shock*	0.1000 (0.0115)	0.1000 (0.0115)	0.0223 (0.0076)
σ_z	*technology shock*	0.0125 (0.0032)	0.0125 (0.0032)	0.0147 (0.0027)
σ_r	*policy shock*	0.0017 (0.0001)	0.0017 (0.0001)	0.0021 (0.0001)
ρ_a	*AR preference shock*	0.5631 (0.0670)	0.5631 (0.0670)	0.9558 (0.0207)
ρ_z	*AR technology shock*	0.9055 (0.0427)	0.9055 (0.0427)	0.9967 (0.0045)
θ_r	*interest rate smoothing*	0.9021 (0.0188)	0.9021 (0.0188)	0.7718 (0.0270)
θ_π	*inflation response*	1.4951 (0.3136)	1.4951 (0.3136)	1.4781 (0.1684)
θ_y	*output response*	0.0904 (0.0467)	0.0904 (0.0467)	-0.0184 (0.0121)
b^a	*aggregate habit*	$0.0000^\dagger$	*0*	0.6016 (0.0668)
b^d	*deep habit*	0.9261 (0.0073)	0.9261 (0.0073)	*0*
χ	*Frisch elasticity*	0.7444 (0.4611)	0.7444 (0.4611)	1.0842 (0.4674)
α	*price adjustment cost*	*29.7919*	*29.7919*	*29.7919*
β	*discount factor*	*0.9965*	*0.9965*	*0.9965*
η	*substitution elasticity*	*6*	*6*	*6*
μ	*markup*	1.2106 (0.0011)	1.2106 (0.0011)	1.2000 (0.0000)
$\ln \mathcal{L}$	*log likelihood*	2375.4246	2375.4246	2341.8950
p-value	*likelihood ratio test*	–	1.0000	0.0000

Notes: The table reports maximum-likelihood estimates of the nested model, the deep habits model ($b^a = 0$), and the aggregate habits model ($b^d = 0$). The price adjustment cost parameter α is set equal to a value that would imply 3-quarter average price contracts in a Calvo-Yun framework. Standard errors are in parentheses. Italicized numbers denote values that are imposed prior to estimation. † denotes an estimate that lies on the theoretical bound of the parameter space.

Table C.2
Parameter Estimates (1965:Q3 - 2012:Q1)

Model Parameter	Parameter Description	Contract Duration 12 months	10 months	8 months	6 months	4 months
σ_a	*preference shock*	0.1229 (0.0170)	0.1071 (0.0131)	0.0939 (0.0104)	0.0857 (0.0100)	0.0829 (0.0113)
σ_z	*technology shock*	0.0162 (0.0059)	0.0133 (0.0038)	0.0118 (0.0028)	0.0110 (0.0023)	0.0104 (0.0019)
σ_r	*policy shock*	0.0017 (0.0001)	0.0017 (0.0001)	0.0017 (0.0001)	0.0017 (0.0001)	0.0017 (0.0001)
ρ_a	*AR preference shock*	0.5012 (0.0719)	0.5400 (0.0692)	0.5879 (0.0641)	0.6350 (0.0566)	0.6665 (0.0504)
ρ_z	*AR technology shock*	0.8983 (0.0449)	0.9050 (0.0426)	0.9040 (0.0437)	0.8947 (0.0477)	0.8830 (0.0513)
θ_r	*interest rate smoothing*	0.9073 (0.0188)	0.9039 (0.0187)	0.9006 (0.0189)	0.8981 (0.0192)	0.8967 (0.0194)
θ_π	*inflation response*	1.4814 (0.3331)	1.4960 (0.3196)	1.4884 (0.3087)	1.4601 (0.3014)	1.4279 (0.2972)
θ_y	*output response*	0.0894 (0.0486)	0.0896 (0.0472)	0.0916 (0.0462)	0.0948 (0.0456)	0.0973 (0.0454)
b^d	*deep habit*	0.9414 (0.0077)	0.9317 (0.0075)	0.9204 (0.0070)	0.9107 (0.0073)	0.9061 (0.0094)
χ	*Frisch elasticity*	1.0655 (0.6334)	0.8776 (0.5199)	0.5984 (0.4024)	0.3279 (0.2901)	0.1645 (0.2101)
α	*price adjustment cost*	*59.3778*	*38.5745*	*22.0936*	*9.9652*	*2.2196*
β	*discount factor*	*0.9965*	*0.9965*	*0.9965*	*0.9965*	*0.9965*
η	*substitution elasticity*	*6*	*6*	*6*	*6*	*6*
μ	*markup*	1.2136 (0.0019)	1.2115 (0.0014)	1.2098 (0.0009)	1.2086 (0.0008)	1.2081 (0.0009)
$\ln \mathcal{L}$	*log likelihood*	2377.6548	2376.1914	2374.6711	2373.3711	2372.5546

Notes: The table reports maximum-likelihood estimates of the deep habits model under alternative settings of the price adjustment cost parameter α. Values of α range from a high of 59.3778, equivalent to 12-month price contracts in a Calvo-Yun framework, to a low of 2.2196, equivalent to 4-month price contracts. Standard errors are in parentheses. Italicized numbers denote values that are imposed prior to estimation.

Table D.1

Parameter Estimates (1965:Q3 - 2012:Q1)

Model Parameter	Parameter Description	Nested Habits Model Unrestricted: $\rho_a \in (-1, 1)$	Restricted: $\rho_a \in [0, 1)$
σ_a	*preference shock*	0.3006 (0.0575)	0.1229 (0.0170)
σ_z	*technology shock*	0.0114 (0.0026)	0.0162 (0.0059)
σ_r	*policy shock*	0.0017 (0.0001)	0.0017 (0.0001)
ρ_a	*AR preference shock*	−0.2999 (0.0922)	0.5011 (0.0719)
ρ_z	*AR technology shock*	0.9333 (0.0295)	0.8983 (0.0449)
θ_r	*interest rate smoothing*	0.9026 (0.0183)	0.9073 (0.0188)
θ_π	*inflation response*	1.5406 (0.3031)	1.4814 (0.3331)
θ_y	*output response*	0.0685 (0.0427)	0.0894 (0.0486)
b^a	*aggregate habit*	0.6111 (0.0546)	0.0000†
b^d	*deep habit*	0.9438 (0.0069)	0.9414 (0.0077)
χ	*Frisch elasticity*	2.0135 (0.6603)	1.0655 (0.6334)
α	*price adjustment cost*	*59.3778*	*59.3778*
β	*discount factor*	*0.9965*	*0.9965*
η	*substitution elasticity*	*6*	*6*
μ	*markup*	1.2142 (0.0019)	1.2136 (0.0019)
$\ln \mathcal{L}$	*log likelihood*	2380.4089	2377.6548

Notes: The table reports maximum-likelihood estimates of the nested habits model under the restriction $\rho_a \in [0, 1)$. Standard errors are in parentheses. Italicized numbers denote values that are imposed prior to estimation. † denotes an estimate that lies on the theoretical bound of the parameter space.

Table E.1

Standard Deviations

Model	SD($\hat{c}_t$)		SD($\hat{\pi}_t$)		SD($\hat{R}_t$)	
Nested Habits	0.0329		0.0083		0.0079	
Aggregate Habits	0.1950		0.0081		0.0081	
VAR(4)	0.0376	[0.0296, 0.0841]	0.0066	[0.0056, 0.0136]	0.0076	[0.0061, 0.0182]

Notes: Simulations of the nested habits model use the parameter values reported in the first column of Table 1. Simulations of the aggregate habits model use the parameter values reported in the third column of Table 1. Numbers in squared brackets correspond to 90% confidence intervals for the standard deviations implied by an unconstrained VAR(4) on $\hat{c}_t$, $\hat{\pi}_t$, and $\hat{R}_t$.

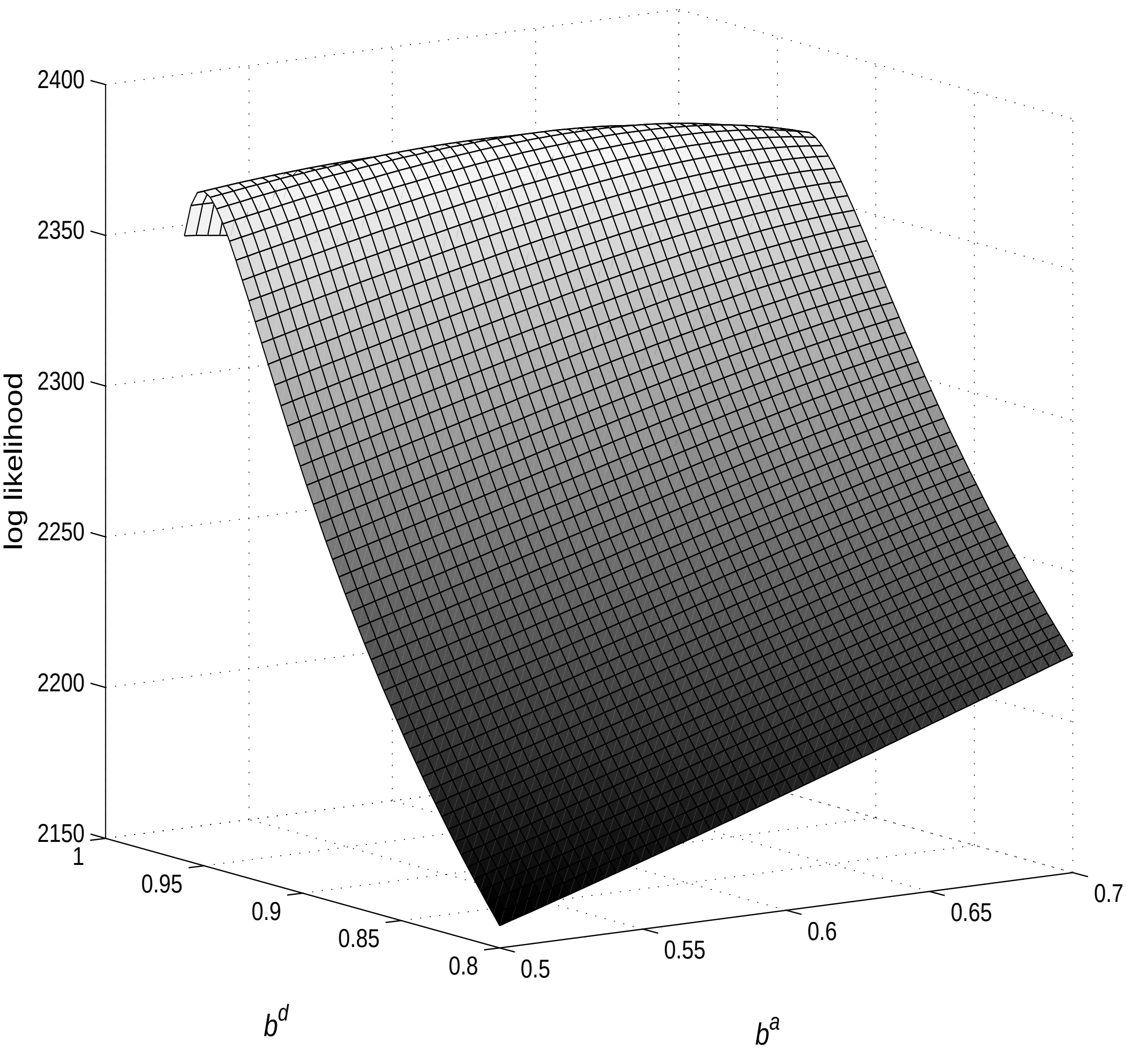

Fig. B.1 A surface plot of the log likelihood function corresponding to the nested habits model is constructed for a range of values of b^d and b^a. All other parameters are held fixed at the maximum-likelihood estimates reported in Table 1.

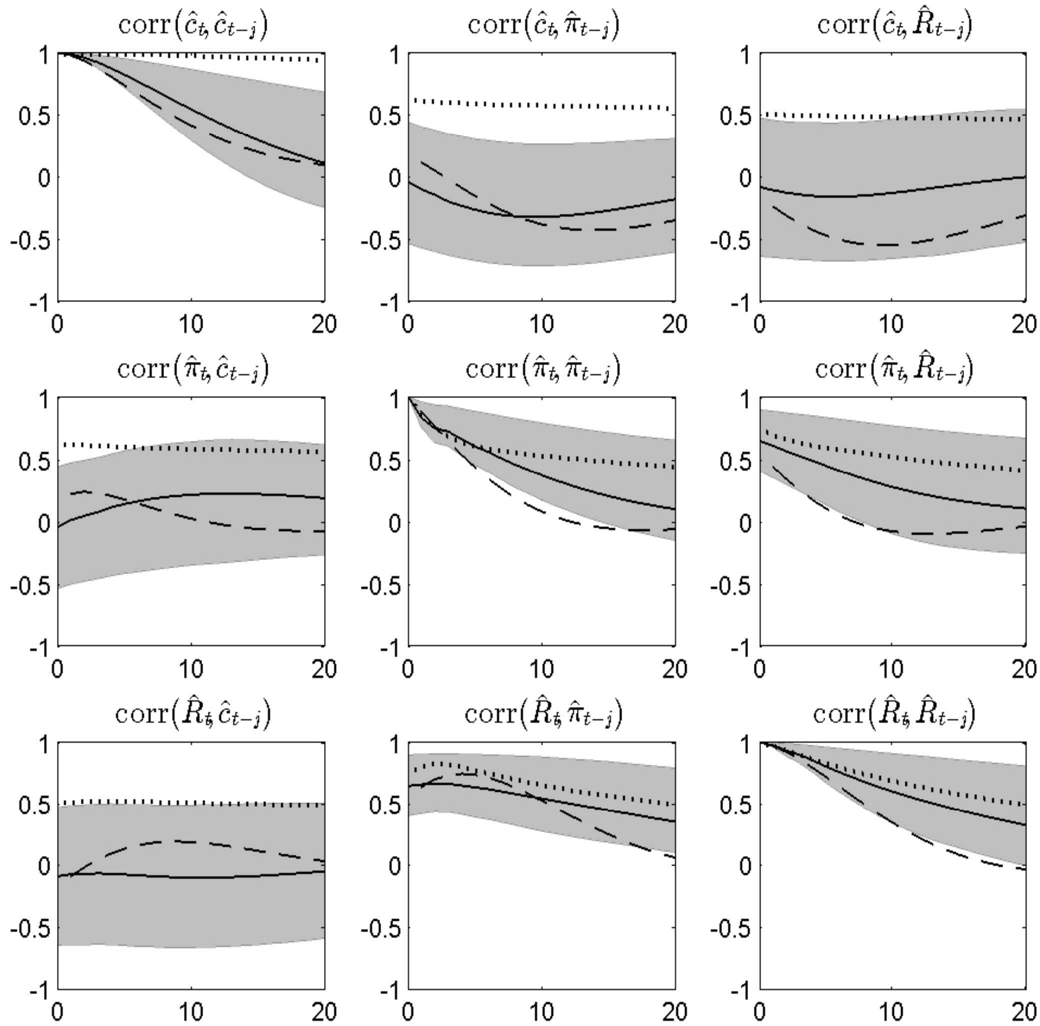

Fig. E.1 The autocorrelation function for consumption $\hat{c}_t$, inflation $\hat{\pi}_t$, and the interest rate $\hat{R}_t$ is drawn for the US data (solid line), the estimated nested habits model (dashed line), and the estimated aggregate habits model (dotted line). Correlations for the US data are obtained from a VAR(4), and the shaded areas correspond to 90% confidence bands.